Math 132 | Fall 2017
Group Quiz 3 | 9/20/17 | Name (Print): Key

For problems 1 - 3, for each function $f(x)$ and value $x = a$ find $\lim_{x\to a^-} f(x)$, $\lim_{x\to a^+} f(x)$, and $\lim_{x\to a} f(x)$ if it exists.

1. Let $f(x) = \tan(x)$ and $g(x) = 1 - x$. Show there exists $c \in (-\frac{\pi}{2}, \frac{\pi}{2})$ so that $f(c) = g(c)$.

$F = f - g.$

$F(0) = 0 - 1 = -1$

$F(\pi/4) = 1 - (1 - \pi/4) = \pi/4$

$\therefore$ $c \in (0, \frac{\pi}{4})$ so that $F(c) = 0 \Rightarrow f(c) = g(c).$

2. Let f be defined as

$$f(x) = \begin{cases} ax^2 - 2x, & x \le 3 \\ 4a \quad x, & x > 3 \end{cases}$$

then find a such that f is continuous.

(set limits =)

$\lim_{x\to 3^-} f(x) = a \cdot 3^2 - 2 \cdot 3 = a9 - 6$

$\lim_{x\to 3^+} f(x) = 4 \cdot a - 3$

$4a - 3 = 9a - 6$

$3 = 5a$

$\boxed{\therefore \ a = \frac{3}{5}.}$

3. Find the tangent of $f(x) = x^2 + 2\sqrt{x}$ at $x = 2$

$f'(x) = 2x + 2\,\frac{1}{2}\,x^{-\frac{1}{2}} = 2x + x^{-\frac{1}{2}}$

4. Using the limit definition of derivative, find $f'(x)$ for $f(x) = \frac{1}{(1+x)^2}$

$$\lim_{h\to 0} \frac{\frac{1}{(1+x+h)^2} - \frac{1}{(1+x)^2}}{h} = \frac{1}{h}\left(\frac{(1+x)^2 - (1+x+h)^2}{(1+x+h)^2(1+x)^2}\right.$$

$$= \frac{1}{h}\,\frac{(1+x)^2 - (1+x)^2 - 2(1+x)h - h^2}{(1+x+h)^2(1+x)^2}$$

$$= \frac{-2 - h}{(1+x+h)^2(1+x)}$$