

1. Suppose a spigot is pouring into a 100 liter tank at a rate of  $r(t) = 100t^{1/2}$  liters per minute. How long does it take to fill the tank?

$$\begin{aligned} \text{(i)} \quad V(t) &= \int_0^t r(s) \, ds \\ &= \int_0^t 100s^{1/2} \, ds \\ &= \frac{200}{3} s^{3/2} \Big|_0^t \\ &= \frac{200}{3} t^{3/2} \end{aligned}$$

$$100 = \frac{200}{3} t^{3/2}$$

$$3/2 = t^{3/2}$$

$$\left(\frac{3}{2}\right)^{2/3} = t$$

$$\text{(ii)} \quad V'(t) = r(t); \quad V(0) = 0$$

$$V(t) = \frac{200}{3} t^{3/2} + C \rightarrow V(0) = C = 0$$

$$V(t) = \frac{200}{3} t^{3/2} = 100$$

$$t = \left(\frac{3}{2}\right)^{2/3} \text{ minutes}$$

2. Let  $F(x) = \frac{1}{x} \int_0^x (10 - 2u) \, du$  on  $0 < x < 5$ . Find all  $c$  satisfying the mean value theorem.

$$\begin{aligned} F(5) &= \frac{1}{5} \int_0^5 (10 - 2u) \, du \\ &= 10 - \frac{1}{5} u^2 \Big|_0^5 \\ &= 5 \end{aligned}$$

$$10 - 2c = 5 = F(5).$$

$$c = 5/2.$$