4.5a Problems

Question 1. Evaluate

(a) $\int (2+3x)^8 \, dx$

$u = 2+3x$; $du = 3dx \leftrightarrow \frac{1}{3}du = dx$

$\frac{1}{9 \cdot 3}(2+3x)^9 + C$

(b) $\int \sin x \cos x \, dx$

(i) $u = \cos x$

$du = -\sin x dx$

$\int -u\,du = -\frac{1}{2}u^2 + C$

$= -\frac{1}{2}\cos^2 x + C$

(ii) $u = \sin x$

$du = \cos x$

$\int u\,du = \frac{1}{2}u^2 + C$

$= \frac{1}{2}\sin^2 x + C$

(c) $\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$

$u = 1 + \tan t$

$du = \sec^2 t \, dt = \frac{1}{\cos^2 t} dt$

$\int \frac{1}{\sqrt{u}} du = 2\sqrt{u} + C = 2\sqrt{1+\tan t} + C$.

Question 2. Evaluate

(a) $\int x^2\sqrt{x+2}\,dx$ $\quad u = x+2 \; ; \; x = u-2 \; ; \; dx = du$

$$= \int (u-2)^2 \sqrt{u}\, du = \int (u^2 - 4u + 4)\sqrt{u} = \int u^{5/2} - 4u^{3/2} + 4u^{1/2}$$

$$= \frac{2}{7} u^{\frac{7}{2}} - 4 \cdot \frac{2}{5} u^{5/2} + 4\,\frac{2}{3} u^{3/2} + C$$

(b) $\int x^3\sqrt{x^2+1}\,dx$ $\quad u = x^2+1 \; ; \; du = 2x \; ; \; (u-1) = x^2$

$$= \frac{1}{2}\int (2x) x^2 \sqrt{x^2+1}\, dx = \frac{1}{2}\int (u-1)\sqrt{u}\, du$$

$$= \frac{1}{2}\int u^{3/2} - u^{1/2}\, du = \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C .$$

(c) $\int \sin t(1-\sin^2 t)^2\,dt$ $\quad$ (Hint: Trigonometric properties FTW)

$$= \int \sin t \cos^4 t\, dt \; ; \quad u = \cos t \rightarrow du = -\sin t\, dt$$

$$= -\int u^4\, du = -\frac{1}{5}u^5 + C = -\frac{1}{5}(\cos t)^5 + C.$$

Question 3. Find the net area under the curve $2+\cos(\pi t/2)$ between $x=0$ and $x=3$

$u = \frac{\pi}{2}t$

$du = \frac{\pi}{2}dt$

$t=0 \to u=0$

$t=3 \to u=\frac{3}{2}\pi$

$$\int_0^3 \left\{2 + \cos\left(\frac{\pi}{2}t\right)\right\} dt$$

$$= \int_0^3 2\,dt + \int_0^{\frac{3}{2}\pi} \frac{2}{\pi}(\cos u)\,du$$

$$= 6 + \frac{2}{\pi}\left(-\sin u \Big|_0^{\frac{3}{2}\pi}\right)$$

$$= 6 + \frac{2}{\pi}$$

Question 4. Calculate

(a) $\displaystyle\int_{1/2}^{1} \frac{\sin(x^{-2})}{x^3}\,dx$ $\displaystyle = \int_4^1 (\sin u)\frac{du}{-2} = \frac{1}{2}\int_1^4 \sin u\,du$

$u = x^{-2}$

$du = -2x^{-3}$

$x = \frac{1}{2} \to u = 4$

$x = 1 \to u = 1$

$$= \frac{1}{2}\left(-\cos u\Big|_1^4\right)$$

$$= \frac{1}{2}\left(\cos(1) - \cos(4)\right)$$

(b) $\displaystyle\int_{-2}^{2} (x+3)\sqrt{4-x^2}\,dx$ (Hint: Algebra then Geometry FTW)

$\cos^2 u = \frac{\cos 2u + 1}{2}$

$2\sin u = x$

$2\cos u\,du = dx$

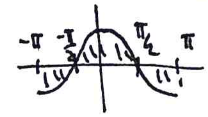
$x=-2 \to u = -\frac{\pi}{2}$

$x=2 \to u = \frac{\pi}{2}$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2\sin u + 3)\,4\cos^2 u\,du$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 8\sin u\cos^2 u\,du + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 12\cos^2 u\,du$$

$$= \frac{8}{3}\cos^3 u\Big|_{-\pi/2}^{\pi/2} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6\,du + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 6\cos 2u\,du$$


$\to 0$

$$\boxed{= 6\pi}$$

Question 5. If f is continuous $\int_0^4 f(x)\,dx = 10$, then $\int_0^2 f(2x)\,dx$ $= \int_0^4 f(u)\,\frac{du}{2} = \frac{10}{2} = 5$

$u = 2x$
$du = 2\,dx$

A. 40

B. 20

C. 10

D. 5

E. None of the above

Question 6. If f is continuous $\int_0^9 f(x)\,dx = 4$, then $\int_0^3 xf(x^2)\,dx$ $= \int_0^9 f(u)\,\frac{du}{2}. = \frac{4}{2}.$

$u = x^2$
$du = 2x\,dx$

A. 8

B. 4

C. 2

D. 1

E. None of the above