3.9 Problems

Standard Problems

Example 1. Find the most general antiderivative of the function:

(a) $f(x) = \sqrt{x^3} + \sqrt[3]{x^2}$
Hint: write radicals as powers

$$f = x^{3/2} + x^{\frac{2}{3}}$$

$$F = \frac{2}{5}x^{\frac{5}{2}} + \frac{3}{5}x^{\frac{5}{3}} + C$$

(b) $f(x) = \frac{x+x^2}{\sqrt{x}}$

$$f = x^{1/2} + x^{3/2}$$

$$F = \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

Example 2. Find the antiderivative so that $f(2) = 1$ Challenging!!

(a) $f(x) = \frac{x}{\sqrt{x-1}} = \frac{x-1+1}{\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}} + \frac{1}{\sqrt{x-1}} = \sqrt{x-1} + (x-1)^{-\frac{1}{2}}$

$$F = \frac{2}{3}(x-1)^{\frac{3}{2}} + 2(x-1)^{\frac{1}{2}} + C$$

$$F(2) = \frac{2}{3} + 2 + C = 1$$

$$C = \frac{5}{3}$$

$$F = \frac{2}{3}(x-1)^{3/2} + 2(x-1)^{1/2} + \frac{5}{3}$$

(b) $f(x) = \frac{x}{\sqrt{x^2-1}}$

$$F = \sqrt{x^2-1} = (x^2-1)^{1/2} + C$$

$$F' = \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x) \quad \checkmark$$

$$F(2) = (4-1)^{1/2} + C = 1$$

$$C = 1 - \sqrt{3}$$

$$F = \sqrt{x^2-1} + 1 - \sqrt{3}$$

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Example 1. Find the most general antiderivative of the function:

(a) $g(x) = \sin x + \frac{1}{2}\cos x$

$G = -\cos x + \frac{1}{2}\sin x + C$

(b) $g(x) = 3\sec^2 x$

$G = 3\tan x + C$

(c) $g(t) = 4\csc^2 x$

$G = -4\cot x + C$

Example 2. Find the most general antiderivative of the function:

(a) $g(t) = -3\sin 2x$

$G = +\frac{3}{2}\cot 2x + C$

(b) $g(t) = x\cos(x^2)$

$G = \frac{1}{2}\sin(x^2) + C$

3.9 Problems

Standard Problems

Example 1. Find the most general antiderivative of the function:

(a) $h(x) = 2(x+1)$

$H = (x+1)^2 + C$

(b) $h(x) = 4x(x^2+1)$

$H = (x^2+1)^2 + C$

(c) $h(t) = x^2(x^3+4) + x^3(x^4+10)$

$H = \frac{1}{6}(x^3+4)^2 + \frac{1}{8}(x^4+10)^2 + C$

Example 2. Find the antiderivative $H(x)$ so that $H(0) = 1$

(a) $h(x) = x\sqrt{x^2+1}$

$$H = \frac{1}{3}(x^2+1)^{\frac{3}{2}} + C$$

$$H(0) = \frac{1}{3} + C = 1.$$

$$C = \frac{2}{3}$$

$$H = \frac{1}{3}(x^2+1)^{\frac{3}{2}} + \frac{2}{3}.$$

(b) $h(t) = 2\sin x(\cos x)^2$

$$H = \frac{2}{3}(\cos x)^3 + C$$

$$H(0) = \frac{2}{3} + C = 1.$$

$$C = \frac{1}{3}$$

$$H = \frac{2}{3}(\cos x)^3 + \frac{1}{3}.$$

3.9 Problems

Standard Problems

Example 1. Find the most general antiderivative of the function:

(a) $k(x) = 3x^2x^5 + 5x^3x^4$

$K = x^3 \cdot x^5 = x^8 + C$

$K' = 3x^2 \cdot x^5 + 5x^3 \cdot x^4$

(b) $k(x) = 2(x+1)(x+3)^2 + 2(x+1)^2(x+3)$

$K(x) = (x+1)^2(x+3)^2 + C$

(c) $k(x) = (x+1)^2 + 2(x+1)(x+3)$

$K(x) = (x+1)^2(x+3) + C$

Example 2. Find the most general antiderivative of the function: Challenging!

(a) $k(x) = \cos^2 x - \sin^2 x$

$K = \cos x \sin x + C$

(b) $k(x) = 2\sqrt{x+1} + \frac{x}{\sqrt{x+1}}$

$K = 2x\sqrt{x+1} + C$

(c) $2x \cos x^2 \sin x + \sin x^2 \cos x$

$K = (\sin x^2)(\sin x) + C$