3.7 Problems (B)

Example 1. A baseball team plays in a stadium that holds 55,000 spectators. With ticket prices at \$10, the average attendance had been 27,000. When ticket prices were lowered to \$8, the average attendance rose to 33,000.

(a) Find the demand function, assuming that it is linear.

$x \equiv$ ticket price

$y \equiv$ # of spectators. (in thousands)

$$\frac{y-27}{x-10} = \frac{33-27}{8-10} = -\frac{6}{2} = -3$$

(b) How should ticket prices be set to maximize revenue?

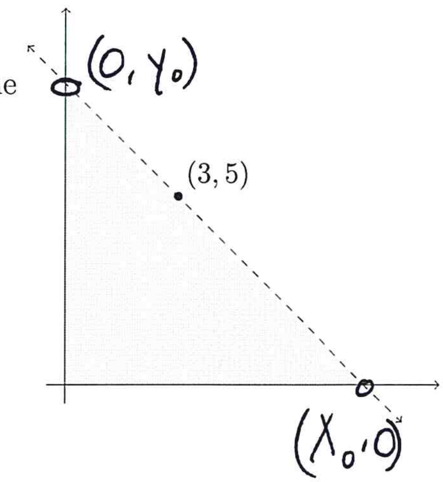
$R \equiv$ revenue. (in thousands).

$$R = xy = x\left(27 - 3(x-10)\right)$$
$$= x(27 - 3x + 30)$$

$$R' = 57 - 6x$$
$$R'' = -6.$$
$$R' = 0 \Leftrightarrow x = \frac{57}{6} = 9.5. \quad \text{local max.}$$

Example 2.

Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.



$$\frac{y-y_0}{x-0} = \frac{5-y_0}{3-0} = \frac{0-y_0}{x_0-0}$$

$$5-y_0 = -3\,\frac{y_0}{x_0}$$

$$x_0 = \frac{3y_0}{y_0-5}.$$

$$A = \frac{1}{2}x_0 y_0 = \frac{3}{2}\,\frac{y_0^2}{y_0-5}$$

$$A' = \frac{3}{2}\,\frac{y_0^2-10y_0}{y_0-5}$$

$$A' = 0 \longrightarrow y_0 = 10$$

y_0 is the min since $y_0 > 5$

and $A' \begin{cases} >0 & \text{for } y_0 > 10 \\ <0 & \text{for } 5 < y_0 < 10. \end{cases}$

min value

$$A(y_0=10) = \frac{3}{2}\,\frac{10^2}{10-5} = 30.$$