3.7 Problems

Example 1. What is the maximum vertical distance between the line $y = x + 2$ and the parabola $y = x^2$ for $x \in [-1, 2]$.

$x^2 < x+2$ for $x \in (-1, 2)$.

$\therefore$ $V(x) \equiv$ vertical distance from $x+2$ to x^2 = $\cancel{x+2-x^2}$ $x+2-x^2$

$V' = 1 - 2x$.

$V'' = -2$.

max @ x so that $V' = 0$

$\Leftrightarrow$

$x = 1/2$.

min distance is

$V(1/2) = 2 + \frac{1}{4}$.

Example 2. What is the minimum vertical distance between the line $y = x - 2$ and the parabola $y = x^2$.

$x^2 > x - 2$ for $x \in \mathbb{R}$.

$V(x) = x^2 - (x-2) = x^2 - x + 2$.

$V'(x) = 2x - 1$

$V''(x) = 2 > 0$

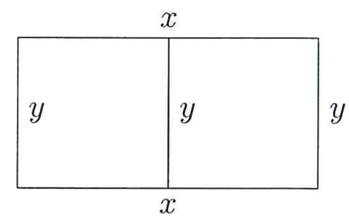
min @ x so that $V' = 0$

$\Leftrightarrow$

$x = 1/2$

$V\left(\frac{1}{2}\right) = 2 - \frac{1}{4}$.

Example 3.



A farmer with 300 ft of fencing wants to enclose a rectangular area and then divide it into two pens with fencing parallel to one side of the rectangle (see the picture to the right).

(a) Write an expression for the total area of the two pens.

$$A = xy$$

(b) Use the given information to write an equation that relates the variables.

$$300 = 3y + 2x$$

(c) Use part (b) to write the total area as a function of one variable.

$$100 - \frac{2}{3}x = y$$

$$A = 100x - \frac{2}{3}x^2$$

(d) Find the largest possible total area of the two pens. Prove that it is the largest.

$$A' = 100 - \frac{4}{3}x$$

$$A'' = -\frac{4}{3} < 0$$

$\therefore$ $A'(x) = 0 \Leftrightarrow$ x is absolute max.

$$A' = 0 \Leftrightarrow x = 75.$$

$$A(x) = 75 \cdot 50 = 3750$$

Example 4. I get my sandwiches toasted at Subway®. The temperature of my sandwich is well approximated by the curve $T(t) = 20 + \dfrac{800t}{t^2 + t + 4}$ where at $t = 0$ my sandwich starts getting toasted.

(a) What is the initial temperature of my sandwich?

$$T(0) = 20$$

(b) How long did my sandwich get toasted for?

Sandwich is done toasting when it meets max temp.

$$T' = 800\left(\frac{t^2+t+4 - t(2t+1)}{(t^2+t+4)^2}\right) = 800\left(\frac{-t^2+4}{(t^2+t+4)^2}\right)$$

$$T' = 0 \Leftrightarrow t = \pm 2.$$

$$T' \begin{cases} > 0 & t \in (0, 2) \\ < 0 & t \in (2, \infty) \end{cases}$$

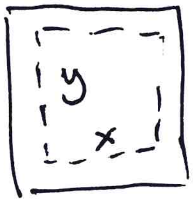
max temp happens at time

$t = 2$.

(c) What is the maximum temperature my sandwich achieved?

$$T(2) = 20 + 800 \frac{2}{2^2+2+4}$$

Example 5. A poster is to have a total area of 180 in^2 with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will have the largest printed area?



$P = xy \equiv$ printed area

$A = (x+2)(y+3) \equiv$ total area $= 180$.

solve for y: $y = \frac{180}{x+2} - 3$

$P = 180 \frac{x}{x+2} - 3x$; $P' = 360 \frac{1}{(x+2)^2} - 3$

$P' = 0 \Leftrightarrow \quad x = \sqrt{120} - 2$.

$y = \frac{180}{\sqrt{120}} - 3$

Example 6. At which point(s) on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have the largest slope?

Slope of tangent line is value of derivative.

$m = y' = 120x^2 - 15x^4$

$m' = 240x - 60x^3$

$= 60x(4 - x^2) = 60x(2-x)(2+x)$

Critical points: $x = 0, \pm 2$.

value of m at these points:

$m(-2) = m(2) = 120(4) - 15(4)^2 = 240$

$m(0) = 0$ largest slope @ $x = \pm 2$.