3.3 Problems Part 1

Derivatives and Graphs

Example 1. For the following functions, find the intervals on which it is increasing and decreasing, and find where any local maximum and/or minimum values occur.

(a) $f(x) = 4x^3 + 3x^2 - 6x + 1$

$f' = 12x^2 + 6x - 6$

$f' = 0 \Rightarrow x = \dfrac{-1 \pm \sqrt{1+8}}{4} = -1, \frac{1}{2}$

$f' > 0$ on $(-\infty, -1) \cup (\frac{1}{2}, \infty) \leftrightarrow f$ increasing

$f' < 0$ on $(-1, \frac{1}{2}) \leftrightarrow f$ decreasing.

Local max @ $x = -1$

Local min @ $x = \frac{1}{2}$.

(b) $g(x) = \sin x + \cos x$ on the domain $[0, 2\pi]$

$g' = \cos x - \sin x$

$g' = 0 \leftrightarrow \tan x = 1 \leftrightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

$g' < 0 \leftrightarrow \tan x > \leftrightarrow x \in (\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{5\pi}{4}, \frac{3}{2}\pi)$

$g' > 0 \leftrightarrow \tan x < \leftrightarrow x \in (0, \frac{\pi}{4}) \cup (\frac{\pi}{2}, \frac{5\pi}{4}) \cup (\frac{3}{2}\pi, 2\pi)$

+ critical points − $x = \frac{\pi}{2}, \frac{3\pi}{2}$

Local max: $\frac{\pi}{4}, \frac{5\pi}{4}, 2\pi$

Local min: $0, \frac{\pi}{2}, \frac{3}{2}\pi$.

(c) $h(x) = x^{2/3}(6 - x)^{1/3}$

$h' = \frac{2}{3}x^{-\frac{1}{3}}(6-x)^{\frac{1}{3}} + \frac{1}{3}x^{2/3}(6-x)^{-\frac{2}{3}}(-1).$

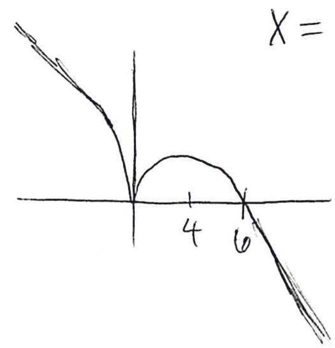
$= \dfrac{\frac{2}{3}(6-x) - \frac{1}{3}x}{x^{1/3}(6-x)^{2/3}} = 0$

$\leftrightarrow h' = 4 - x = 0$

$x = 4.$

local max @ $x = 4$

local min @ $x = 0$



Example 2. Again consider the function: $f(x) = 4x^3 + 3x^2 - 6x + 1$

(a) Use the concavity test to find the intervals of concavity and the inflection points.

$f'' = 24x + 6$

$f'' > 0 \iff x > -\frac{1}{4}$

$f'' < 0 \iff x < -\frac{1}{4}$

(b) Use your results above, along with the facts that

- f has a y-intercept at $y = 1$
- f has x-intercepts near $x = -1.7, 0.2,$ and 0.8.

to sketch the graph of the function.