# 2.4b - 2.5a Problems

## Trig Derivatives

**Example 1.** Calculate the derivatives:

(a) $\frac{d}{dx}(\cos(x)\sin(x))$ Product Rule: $(fg)' = f'g + fg'$

$$(\cos x \sin x)' = (\cos x)' \sin x + \cos x (\sin x)'$$

$$= -\sin^2 x + \cos^2 x$$

(b) $\frac{d}{dx}\left(\frac{1}{\cos(x)\sin(x)}\right)$ Quotient Rule: $\frac{f}{g} = \frac{gf' - fg'}{g^2}$

$$(1)' = 0$$

$\therefore$

$$\left(\frac{1}{\cos x \sin x}\right)' = \frac{-(\cos x \sin x)'}{(\cos x \sin x)'}$$

$$= \frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x}$$

**Example 2.** Evaluate the trig derivatives:

(a) Let $f(x) = 5x\cot x$, find $f'\left(\frac{2\pi}{3}\right)$ ; $f(x) = 5x\left(\frac{\cos x}{\sin x}\right)$

$$(\cot x)' = \frac{\sin x\,(\cos x)' - \cos x\,(\sin x)'}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

$$= -\csc x$$

$$(5x\cot x)' = 5\cot x + 5x(-\csc^2 x)$$

$$f'\left(\frac{2\pi}{3}\right) = 5\,\frac{1/2}{\sqrt{3}/2} + 5\,\frac{2\pi}{3}\left(-\frac{1}{(\sqrt{3}/2)^2}\right)$$

$$= 5\,\frac{1}{\sqrt{3}} - 5\,\frac{8\pi}{9}$$

(b) Let $f(x) = 5\tan x$, find $\cancel{f'\left(\frac{3\pi}{2}\right)}$ $f'\left(\frac{2\pi}{3}\right)$

$$f'(x) = 5\,\frac{\cos x\,(\sin x)' - \sin x\,(\cos x)'}{\cos^2 x}$$

$$= 5\,\frac{1}{\cos^2 x} = 5\sec^2 x$$

$$f'\left(\frac{2\pi}{3}\right) = \frac{1}{\cos^2\left(\frac{2\pi}{3}\right)} = \frac{1}{(\sqrt{3}/2)^2} = \frac{4}{3}$$

## The Chain Rule

**Example 3.** Find the derivative of each function

(a) $f(x) = (2x^3 + 5x^2 + 7)^{10}$.

"power rule inside power rule"

$$f'(x) = 10\,(2x^3+5x^2+7)^9\,(2x^3+5x^2+7)'$$

$$= 10\,(2x^3+5x^2+7)^9\,(6x^2+10x).$$

(b) $g(x) = \sqrt[3]{1 + \tan x}$.

"trig inside power rule"

$$g(x) = (1+\tan x)^{1/3}\ ; \quad g'(x) = \tfrac{1}{3}(1+\tan x)^{-2/3}\,(1+\tan x)'$$

$$= \tfrac{1}{3}(1+\tan x)^{-2/3}\,\sec^2 x.$$

(c) $h(x) = 4\sec\left(\frac{x}{8}\right)$.

$$\frac{d}{dy}\sec y = \frac{-\sin y}{\cos^2 y} = -\tan y\,\sec y$$

$$\frac{d}{dx}h(x) = 4\left(-\tan\left(\tfrac{x}{8}\right)\sec\left(\tfrac{x}{8}\right)\right)\left(\tfrac{x}{8}\right)'$$

$$= -\tfrac{1}{2}\tan\left(\tfrac{x}{8}\right)\sec\left(\tfrac{x}{8}\right)$$

**Example 4.** Suppose that $f(x) = \cos^6 x$

(a) find $f'(x)$

$(\cos y)' = -\sin y \qquad (\omega^6)' = 6\omega^5$

$\therefore \quad f'(x) = 6(\cos^5 x)(-\sin x)$

(b) find the equation of the tangent line to the curve $y = f(x)$ at the point where $x = \frac{\pi}{6}$

$f'\left(\frac{\pi}{6}\right) = 6\left(\frac{\sqrt{3}}{2}\right)^5\left(-\frac{1}{2}\right) = -\frac{6 \cdot 3^{5/2}}{2^6}; \quad f\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)^6 = \frac{3^3}{2^6}$

$\therefore$ Equation of tangent line in $(\hat{x}, \hat{y})$

$$\frac{\hat{y} - 3^3/2^6}{\hat{x} - \pi/6} = -6\,\frac{3^{5/2}}{2^6}$$

**Example 5.** Let $f(x) = 3\tan(2\cos(5x))$ find $f'(x)$ 2 applications of Chain Rule

$f' = \left(3\tan(2\cos(5x))\right)' = 3\sec^2(2\cos(5x))\,(2\cos(5x))'$

$= 3\sec^2(2\cos(5x))\,(-2\sin(5x))\,(5x)'$

$= -30\sec^2(2\cos(5x))\sin(5x)$.