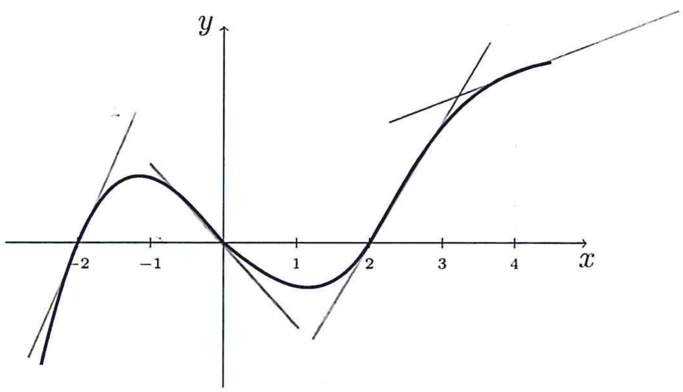
2.1 Problems

Example 1. For the function f whose graph is given to the right, arrange the following numbers in increasing order and explain your reasoning.

$$0 \qquad f'(-2) \qquad f'(0) \qquad f'(2) \qquad f'(4)$$

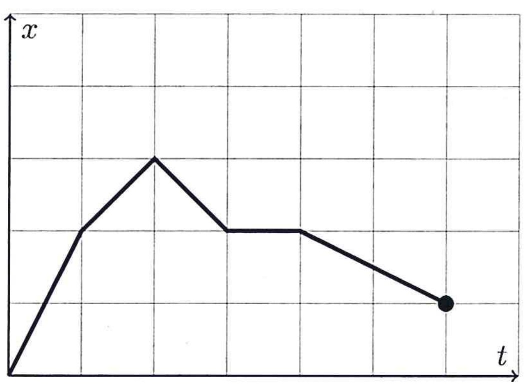


$f'(0) < 0 < f'(4) < f'(2) < f'(-2)$.

Example 2. A particle starts by moving to the right along a horizontal line; the graph of its position function is below for $t \in [0, 6]$ seconds.

(a) When is the particle moving to the right? Moving to the left? Standing still?

(b) Draw a graph of the velocity function.



(a) *Particle moving to right:

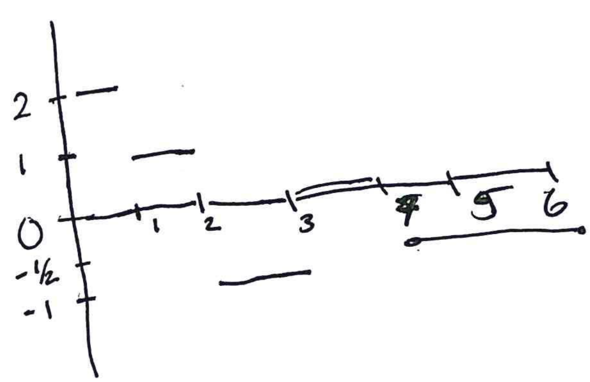
$(0,2)$

*Particle moving to left.

$(2,3) \cup (4,6)$

* Standing still $(3,4)$

(b)



Standard Problems

Example 3. Find $f'(1)$ for $f(x) = 2x^2 - 3x + 5$

$$f' = 4x - 3$$

$$f'(1) = 4(1) - 3 = 1.$$

Example 4. Find the equation of the tangent line for $f(t) = \dfrac{2t+1}{t+3}$ at $t = 4$.

$$f' = \frac{(2t+1)'(t+3) - (2t+1)(t+3)'}{(t+3)^2} = \frac{2(t+3) - (2t+1)}{(t+3)^2}$$

$$f' = \frac{5}{(t+3)^2} \; ; \; f'(4) = \frac{5}{7^2} \; ; \; f(4) = \frac{9}{7}$$

$$\frac{y - 9/7}{x - 4} = \frac{5}{7^2}$$

Example 5. Find the equation of the tangent line for $g(x) = \sqrt{5 - x}$ at $x = 1$.

$$g'(x) = \left((5-x)^{1/2}\right)' = \frac{1}{2}(5-x)^{-\frac{1}{2}}$$

$$g(1) = \sqrt{5-1} = 2 \qquad g'(1) = \frac{1}{4}.$$

$$\frac{y-2}{x-1} = \frac{1}{4}.$$

Non-Standard Problems

Determine whether $f'(0)$ exists.

Example 6. $f(x) = \begin{cases} x\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$$\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(h) - f(0)}{h} = \lim_{h\to 0} \frac{h \sin\frac{1}{h}}{h}$$

$$= \lim_{h\to 0} \sin\left(\frac{1}{h}\right).$$

Does the limit exist?

consider $h = \frac{1}{\frac{\pi}{2} + 2\pi k}$ $k = 1, 2, 3, \ldots$ $\therefore h \to 0$

then $\sin\left(\frac{1}{\frac{\pi}{2}+2\pi k}\right) = 1.$ for all k.

on the other hand

$h = \frac{1}{2\pi k}$ $k = 1, 2, 3, \ldots$ $\therefore h \to 0.$

and $\sin\left(\frac{1}{2\pi k}\right) = 0$ for all k.

So $\lim_{h\to 0} \sin\frac{1}{h}$ DNE

Determine whether $f'(0)$ exists.

Example 7. $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$$\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{\left(h^2 \sin \frac{1}{h}\right) - 0}{h}$$

$$= \lim_{h\to 0} h \sin \frac{1}{h}$$

We find the limit using the squeeze theorem.

* for all h, $\quad -1 \leq \sin \frac{1}{h} \leq 1$

* Thus, $\bullet\ h \sin \frac{1}{h} \leq h \quad$ for $h > 0$.

$\bullet\ h \sin \frac{1}{h} \geq -h$

So $\displaystyle\lim_{h\to 0^+} h \sin \frac{1}{h} = 0$ since $\displaystyle\lim_{h\to 0^+} h = \lim_{h\to 0^+} (-h) = 0$.

A similar demo shows

$$\lim_{h\to 0^-} h \sin \frac{1}{h} = 0$$

So the limit exist & $\underline{\underline{=0}}$