Instructions:
- No electronic devices that can access the web, exchange text, or messages, etc., can be used.
- You must show appropriate legible work and justify your answer to receive full credit.
- There are 30 possible points. Partial credit will be given.
- Check-and-mark your answers as indicated on the answer page.

Good Luck!

ACADEMIC HONOR CODE

As a student and citizen of the Michigan State University Community, I pledge to not lie, cheat, or steal.

Name: ____________________________

Problem | Score | Out of
--- | --- | ---
1 |   | 5
2 |   | 5
3 |   | 7
4 |   | 5
5 |   | 8
Total |   | 30

SIGNED: ____________________________

March 30, 2015
ECE 366, Exam 2
What is the output of the system when \( x(t) = 2 \cos(\pi t) - \frac{3}{2} \) for all \( t \)? Show all work. (5 points)

I. Transfer Function: Suppose that an \( \text{LTI} \) system has the transfer function \( H(s) = \frac{\alpha e^{-s}}{s+\frac{1}{2}} \).

and

\[
\begin{align*}
X(t) \xleftarrow{\text{LTI}} x(t) \quad &\quad \text{for} \quad \alpha e^{-3t} \in \mathbb{R}^-
\end{align*}
\]
\[
\text{below } (5 \text{ points)}:
\text{ 2. Fourier Series: Find the Fourier Series coefficients, } C_n \text{ for the periodic exponential function}
\]
3. Fourier Transforms and Filtering: Consider the following periodic tent function, $x(t)$:

(a) Compute the Fourier Transform of $x(t)$. Show all work, and reference the notes/book as necessary. [4 points]

$g(t) = \text{tent}(\frac{t}{2}) = A(\frac{t}{2})$

$X(\omega) = \frac{A}{\pi} \sin c(\frac{\omega}{2})$

(b) Suppose your boss asks you to design a Low Pass Filter (LPF) whose output when given $x(t)$ is $y(t) = 2$. Graph the frequency response, $H(\omega)$, of the LPF you would design to accomplish this task. Label all important points. [3 points]

$X(\omega) = \pi \sum_{k=-\infty}^{\infty} \text{sin}^2(\frac{\omega}{2}) S(\omega - k\pi)$

$x(t) = \pi T$ and $y(t) = 2 \left\{ \right.$

$H(\omega)$

$\chi(w) = \frac{1}{\omega}$
\[
\begin{bmatrix}
\frac{1}{2} & -1 & 0 \\
-1 & \frac{1}{2} & 0 \\
0 & 0 & 1
\end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix}
2i \sin(\Theta) \\
-1 + 10 \cos(\Theta) \\
-1 - 10 \cos(\Theta)
\end{bmatrix}
\]

So

\[
\left(\frac{1}{2}\right)^2 - 1^2 = \frac{1}{4} - 1 = -\frac{3}{4}
\]

\[
\sin(\Theta) = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{6}
\]

\[
\begin{bmatrix}
\frac{3\pi}{2} \\
\pi \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\begin{bmatrix}
\frac{3\pi}{2} \\
\pi \\
0
\end{bmatrix}
\end{bmatrix}
\]

Show all work. Use at most one property of the Fourier Transform at a time. 5 points.

\[
\frac{ip}{x^p} \sin(\Theta) = (\Theta)
\]

4. Properties of the Fourier Transform. Suppose that the Fourier transform of the signal \(x(t)\) is

\[
\hat{x}(\omega) = \exp(-\pi \omega^2)
\]

Calculate the Fourier transform of the modulated derivative of \(x(t)\).
\[
\begin{align*}
\sqrt{T} &= \frac{1}{\pi} \int_0^{\pi} \frac{dt}{\left( \text{rect}(\frac{t}{\pi}) \right)^2} \\
\text{Thus, } A(t) &= x(t) \text{ is}
\end{align*}
\]

\[x(t) \xrightarrow{\text{rect}} \text{rect}\left( \frac{t}{2} \right) \]

\[\text{So, } x(t) \cdot \text{rect}(\frac{t}{2}) \Rightarrow \text{sinc}(t) = \frac{1}{\pi} \int \frac{\text{rect}(\frac{t}{2})}{\text{sinc}(t)} \, dt \]

\[\text{Use Parseval's Theorem to compute the energy of } x(t) = \text{sinc}(t) \]

\[\text{Show all work.} \]