Math 881 # Homework 4 **Due Wed Feb 7**

You are encouraged to discuss homework problems with other students, but you must write out solutions in your own words. LaTeX is encouraged, but not required. If you get significant help from a reference or person, give explicit credit.

1. Recall that a planar graph is one which can be drawn in the plane without edge crossings: that is, the topological realization $\Delta(G)$ can be embedded in the plane $\mathbb{R}^2$. Prove that the following conditions are equivalent for a planar graph G.

(a) G has n vertices and $m = 3n - 6$ edges

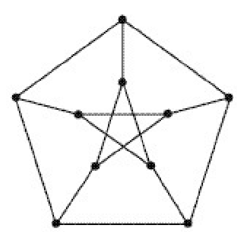
(b) G is maximal planar: $G + xy$ is not planar for any edge $xy \notin E(G)$

(c) G has only triangular faces, including the infinite face: $\deg(F) = 3$ for all F.

In your arguments, you may use results proved in class and intuitive, obvious topological facts.

2. For a graph K, recall that a topological minor TK is obtained by replacing edges of K by independent paths in TK; and for a general minor IK, we may reduce to K by repeatedly contracting an edge $e = xy \in E(G)$ to a vertex v_{xy}.
PROBLEM: Show that if $\deg(v) \leqslant 3$ for all $v \in V(K)$, and $IK \subset G$, then $TK \subset G$. (The case $K = K_{3,3}$ was a lemma for Kuratowski's Theorem.)

3. Consider the Petersen Graph G:



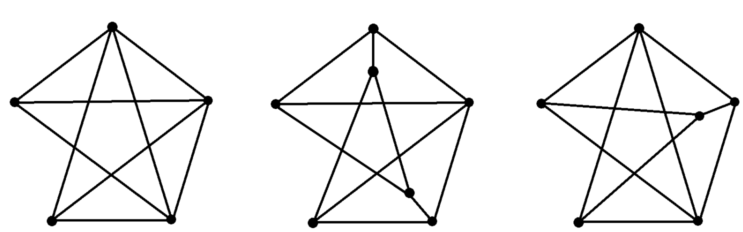
The Petersen Graph.

a. Show that if G were planar, it would violate the Edge-Region Inequality, so it must be non-planar.

Also: Find some graph which would satisfy the Edge-Region Inequality, but which is nevertheless non-planar.

b. Draw G as an IK_5: circle some edges which produce K_5 when contracted. As we dicussed in lecture, use this IK_5 structure to find a TK_5 or $TK_{3,3} \subset G$.

c. In lecture, we gave a planar drawing (with straight-line edges) of the graph $K_5 - e$. Starting with this drawing, perform the Kuratowski algorithm to draw the two graphs G_1, G_2 below, in each case obtaining a straight-line planar drawing or an obstructing TK_5 or $TK_{3,3}$.



K_5 - e G_1 G_2