Additions/Corrections

Calculus of Variations

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Chapter 1

Chapter 2

Exercise 2.35 You and the other traffic are maintaining a constant speed in the right lane of a freeway. You wish to glance ahead by momentarily moving into the left lane. What constant-speed strategy will cost you the least headway? Should you move quickly into the left lane then back, or should you change lanes slowly? Formulate this problem.

Chapter 3

Chapter 4

Exercise 4.54 Contrary to popular belief, the period $T$ of oscillation of the planar pendulum (CVP 8) is not independent of its maximal angle $\theta_0$ of excursion. Larger $\theta_0$ yield larger periods $T$; in the extreme case, $T = \infty$ when $\theta_0 = \pi$.

Prove that

$$
\frac{T}{4} = \sqrt{\frac{a}{2g}} \int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos \theta - \cos \theta_0}} = k \sqrt{\frac{a}{g}} \int_0^{\theta_0/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}.
$$

where $k = \sin(\theta_0/2)$. Prove that in the limit as $\theta_0 \to 0$, the period $T$ approaches the period $2\pi \sqrt{a/g}$ of the harmonic oscillator $\ddot{\theta} = -(g/a)\theta$.

Chapter 5

Exercise 5.39 We have seen paths that are stationary for a cost but do not minimize it — see (3.9). Attempt to formulate the notion that a path $y = y(x)$ makes stationary an integral cost like (5.1a) yet satisfies an integral constraint like (5.1b).
(Daniel Liberzon) The first term in the integrand of (9.46) should be \( L_\eta \eta' \).

**Exercise 9.15** typo: \( x^T A x \geq \gamma |x|^2 = \gamma x^T x \).

**Exercise 9.45** The concepts *weak global minimum, strong global minimum, and minimum* coincide.

Chapter 10

**Chapter 11**

(John Prussing) Corollary B. (Jacobi’s necessary condition)

(Daniel Liberzon) Last term in (11.16) should be \( -(w - z)L_\eta'(x, y, z) \)

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