1 Functions And Change

1.1 What Is a Function?

*** Function**

A **function** is a rule that takes certain numbers as inputs and assigns to each a definite output number. The set of all input numbers is called the **domain** of the function and the set of resulting output numbers is called the **range** of the function.

*** The Rule of Four: Tables, Graphs, Formulas, and Words**

Functions can be represented by tables, graphs, formulas, and descriptions in words.

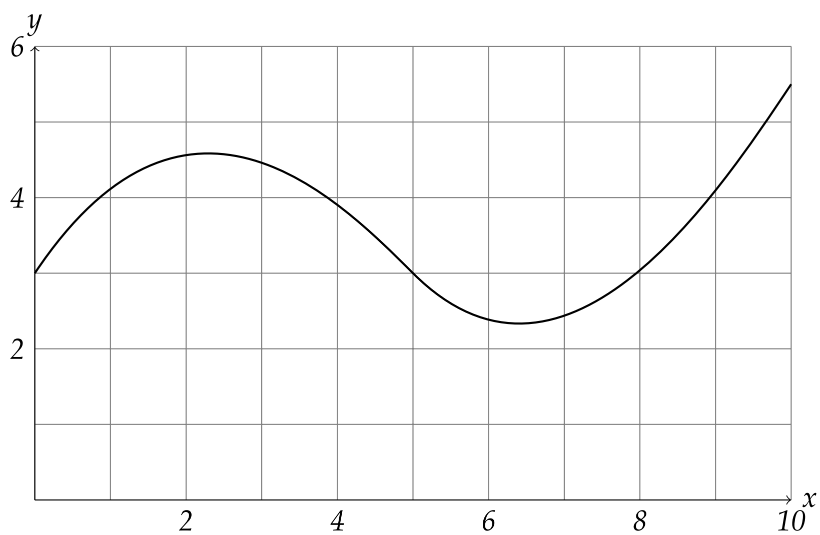
*** Function Notation and Intercepts**

We write $y = f(x)$ to express the fact that y is a function of x. The independent variable is x, the dependent variable is y, and f is the name of the function.
The graph of a function has an **intercept** where it crosses the horizontal or vertical axis.
The **horizontal intercept** is the value of x such that $f(x) = 0$.
The **vertical intercept** is the value of y when $x = 0$, which is given by $f(0)$.

Example 1 *For the function given in the following table, find $f(3)$ and $f(7)$.*

x	1	2	3	4	5	6	7	8
$f(x)$	11.3	11.8	12.2	12.7	13.1	14.0	14.6	15.2

Example 2 *The following graph defines y as a function of x. Find $f(5)$ and $f(10)$.*



Example 3 *Let $y = f(x) = x^2 - 2$.*

(a) What is $f(1)$?

(b) Find the vertical intercept of $f(x)$.

(c) Find the horizontal intercept of $f(x)$.

(d) What values of x give y a value of 14?

(e) Are there any values of x that give y a value of -11?

Example 4 *Let $W = f(t)$ represent wheat production in Argentina, in millions of metric tons, where t is years since 1990.*

(a) Interpret $f(12) = 9$ in terms of wheat production.

(b) What is the meaning of $f(20)$?

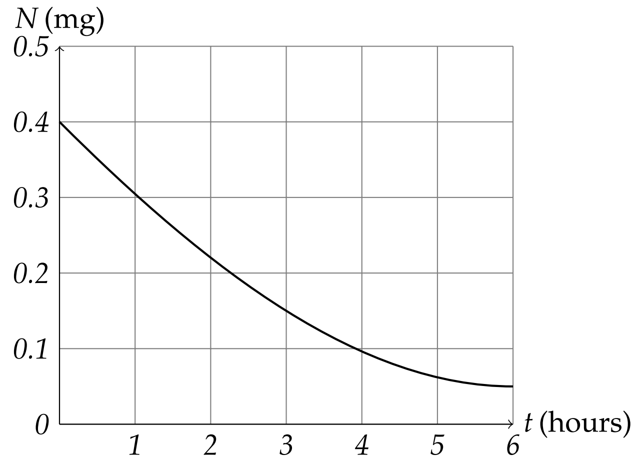
Example 5 *World annual CFC (chlorofluorocarbons) consumption, $C = f(t)$, in million tons, is a function of time, t, in years since 1987. (CFCs are measured by the weight of ozone that they could destroy.)*

(a) Interpret $f(10) = 0.2$ in terms of CFCs.

(b) Interpret the vertical intercept of the graph of this function in terms of CFCs.

(c) Interpret the horizontal intercept of the graph of this function in terms of CFCs.

Example 6 *The following figure shows the amount of nicotine, $N = f(t)$, in mg, in a person's bloodstream as a function of the time, t, in hours, since the person finished smoking a cigarette.*



(a) *Estimate $f(3)$ and interpret it in terms of nicotine.*

(b) *About how many hours have passed before the nicotine level is down to 0.1 mg?*

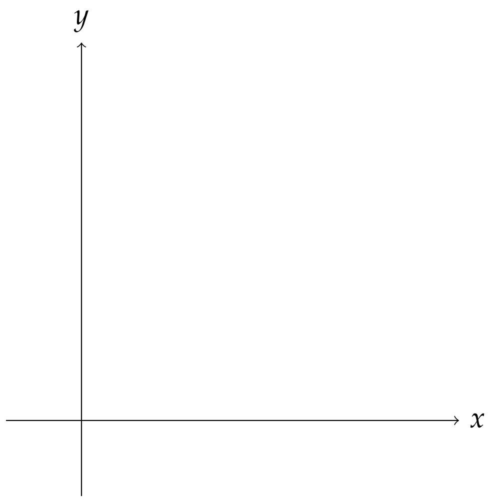
(c) *What is the vertical intercept? What does it represent in terms of nicotine?*

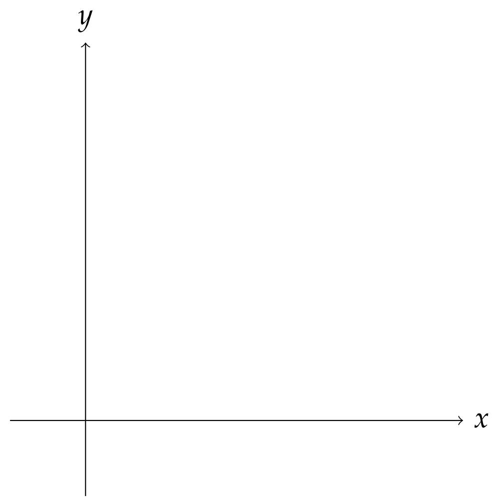
(d) *If this function had a horizontal intercept, what would it represent?*

*** Increasing and Decreasing Functions**

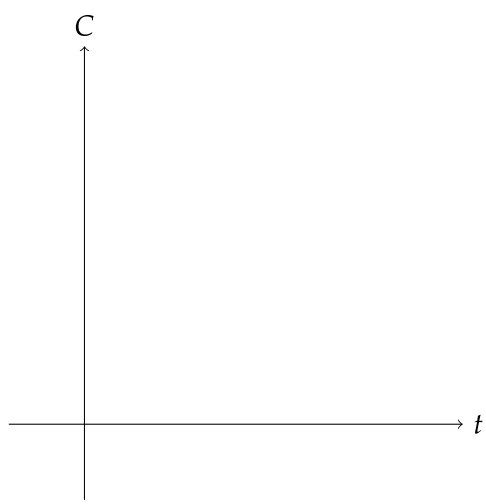
A function f is **increasing** if the values of $f(x)$ increases as x increases.
A function f is **decreasing** if the values of $f(x)$ decreases as x increases.

The graph of an **increasing** function **climbs** as we move from left to right.
The graph of an **decreasing** function **descends** as we move from left to right.





Example 7 *After an injection, the concentration of a drug in a patient's body increases rapidly to a peak and then slowly decreases. Graph the concentration of the drug, C, in the body as a function of the time, t, since the injection was given. Assume that the patient has none of the drug in the body before the injection.*



1.2 Linear Functions

Linear functions are those functions whose graphs are **straight lines**.

*** Slope and Rate of Change**

Given two points (x_1, y_1) and (x_2, y_2) on the graph of a linear function $y = f(x)$. The slope is given by

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Example 1 *Find the slope of the line that passes through* $(0, 2)$ *and* $(2, 3)$.

*** Linear Functions in General**

A **linear function** has the form

$$y = f(x) = b + mx.$$

Its graph is a line such that
(a) m is the **slope**, or the rate of change of y with respect to x.
(b) b is the **vertical intercept** or values of y when x is zero.

If m is positive, then f is increasing. If m is negative, then f is decreasing.

The equation of a line of slope m through the point (x_0, y_0) is

$$y - y_0 = m(x - x_0).$$

Example 2 *Find an equation for the line that passes through* $(4, 5)$ *and* $(2, -1)$.

Example 3 *A city's population was* 30,700 *in the year 2000 and is growing by 850 people a year.*

(a) Give a formula for the city's population, P*, as a function of the number of years,* t*, since 2000.*

(b) What is the population predicted to be in 2012?

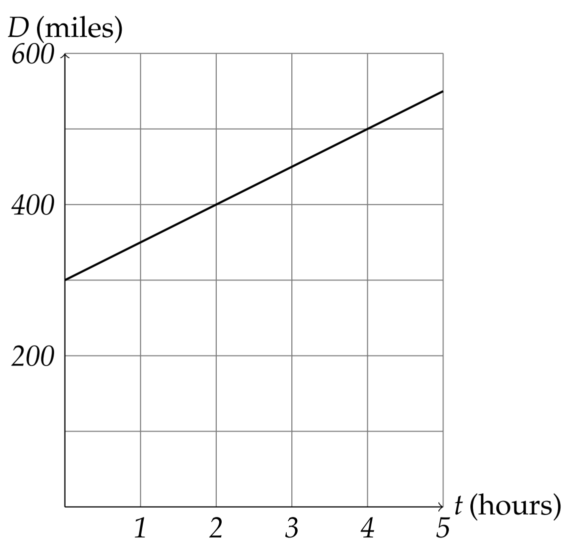
(c) When is the population expected to reach 45,000?

Example 4 *The following figure shows the distance from home, in miles, of a person on a 5-hour trip.*

(a) Estimate the vertical intercept. Give units and interpret it in terms of distances from home.

(b) Estimate the slope of this linear function. Give units, and interpret it in terms of distance from home.

(c) Give a formula for distance, D*, from home as a function of time,* t*, in hours.*



Example 5 *Annual revenue R from McDonald's restaurants worldwide can be estimated by* $R = 19.1 + 1.8t$*, where R is in billions dollars and t is in years since January 1, 2005.*

(a) *What is the slope of this function? Include units. Interpret the slope in terms of McDonald's revenue.*

(b) *What is the vertical intercept of this function? Include units. Interpret the vertical intercept in terms of McDonald's revenue.*

(c) *What annual revenue does the function predict for 2012?*

(d) *When is annual revenue predicted to hit 35 billion dollars.*

Example 6 *World grain production was 1241 million tons in 1975 and 2048 million tons in 2005, and has been increasing at an approximately constant rate.*

(a) *Find a linear function for world grain production, P, in million tons, as a function of t, then number of years since 1975.*

(b) *Interpret the slope in terms of grain production.*

(c) *Interpret the vertical intercept in terms of grain production.*

*** Recognizing Data from a Linear Function**

Values of x and y in a table could come from a **linear** function $y = b + mx$ if the **difference** in y-values are **constant** for equal differences in x.

Example 7 *Which of the following tables of values could represent a linear function? For each table that could represent a linear function, find a formula for that function.*

(a)

x	0	1	2	3
$f(x)$	2.5	3.0	3.5	4.0

(b)

x	0	2	4	6
$g(x)$	100	90	70	40

(c)

t	20	30	40	50
$h(t)$	2.4	2.2	2.0	1.8

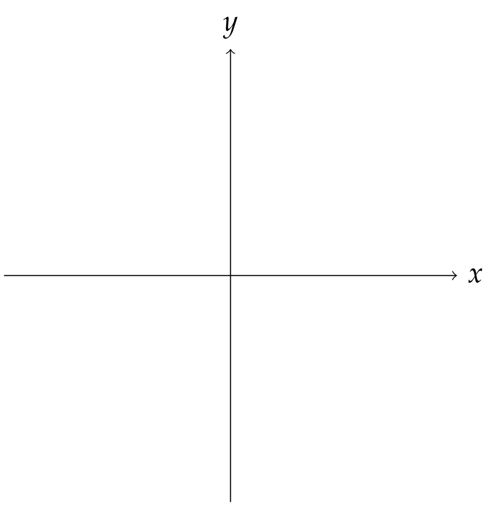
Example 8 *Given the following table, find formulas for each of the following functions.*

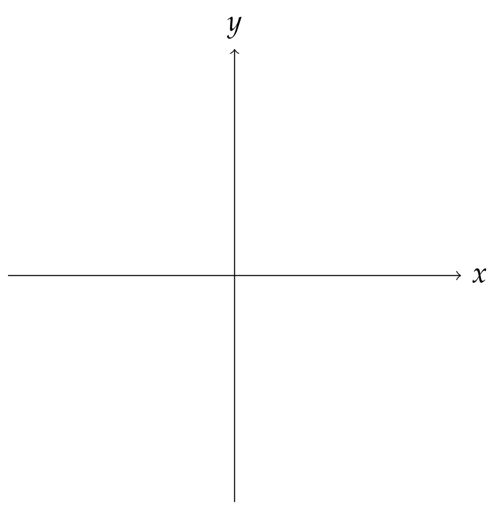
t	15	20	25	30
s	62	72	82	92

(a) *s as a linear function of t.*

(b) *t as a linear function of s.*

*** Families of Linear Functions**





1.3 Average Rate of Change and Relative Change

*** Average Rate of Change**

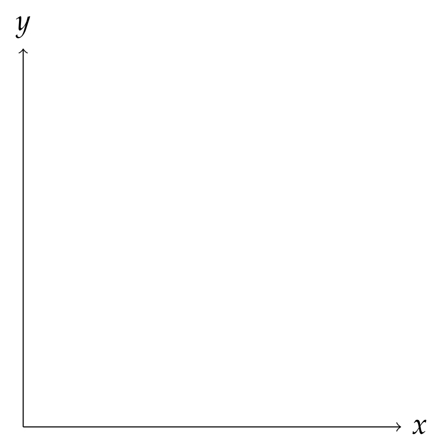
For a function $y = f(x)$, the Average Rate of Change (AROC) of f between $x = a$ and $x = b$ is given by

$$\text{AROC} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}.$$

The units of AROC are units of y per unit of x.

*** Visualizing Rate of Change**

Given $y = f(x)$, the AROC of f between $x = a$ and $x = b$ is the slope of secant line connecting $(a, f(a))$ and $(b, f(b))$.



Example 1 *Find the average rate of change of $f(x) = 3x^2 + 4$ between $x = -2$ and $x = 1$.*

Example 2 *The balance, B, of an account after t years is given by $B = 1000(1.08)^t$. Find the average rate of change in the balance over the interval $t = 0$ to $t = 5$. Give units and interpret your answer in terms of the balance in the account.*

Example 3 *The following table gives the net sales of The Gap, Inc, which operates nearly 3000 clothing stores.*

Year	2003	2004	2005	2006	2007	2008
Sales	15,584	16,267	16,019	15,923	15,763	14,526

(a) Find the change in sales between 2004 and 2008.

(b) Find the average rate of change in sales between 2004 and 2008. Give units and interpret your answer.

(c) From 2003 to 2008, were there any one-year intervals during which the average rate of change was positive? If so, when?

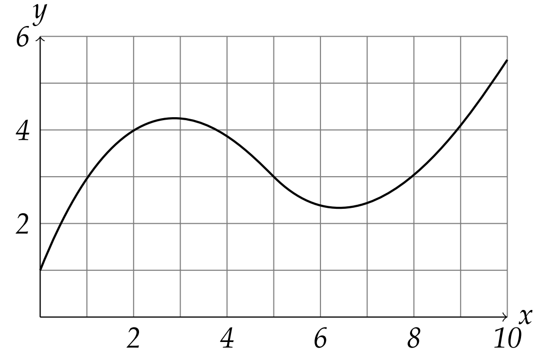
*** Concavity**

The graph of a function is concave up if it bends upward as we move left to right, or the AROC is increasing from left to right.
The graph of a function is concave down if it bends downward as we move left to right, or the AROC is decreasing from left to right.
A line is neither concave up nor concave down.

Example 4 *The figure below shows the graph of $y = f(x)$. Estimate the intervals over which:*

(a) The function is increasing; decreasing.

(b) The graph is concave up; concave down.



Example 5 *The following table gives values of $f(t)$. Is f increasing or decreasing? Is the graph of f concave up or concave down?*

t	0	5	10	15	20	25
$f(t)$	13.1	14.1	16.2	20.0	29.6	42.7

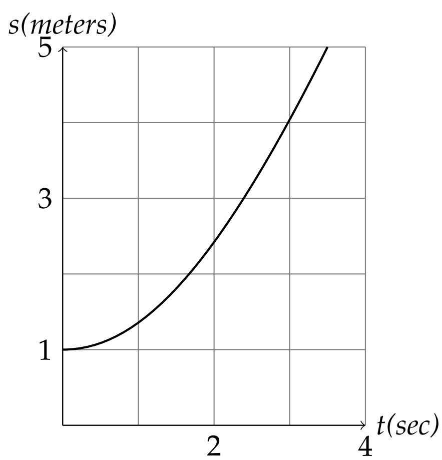
Example 6 *The following table gives values of $g(t)$. Is g increasing or decreasing? Is the graph of g concave up or concave down?*

t	0	2	4	6	8	10
$g(t)$	40	37	33	28	22	15

*** Average Velocity**

$$\boxed{\text{Average velocity} = \frac{\text{Change in distance}}{\text{Change in time}}.}$$

Example 7 *The following figure shows a particle's distance, s, from a point. What is the particle's average velocity from $t = 0$ to $t = 3$?*



Example 8 *At time t in seconds, a particle's distance $s(t)$, in cm, from a point is given in the table. What is the average velocity of the particle from $t = 6$ to $t = 13$?*

t	0	3	6	10	13
$s(t)$	0	72	92	144	180

Example 9 *The following figure shows the position of an object at time t.*

(a) *Draw a line on the graph whose slope represents the average velocity between $t = 2$ and $t = 8$.*

(b) *Is average velocity greater between $t = 0$ and $t = 3$ or between $t = 0$ and $t = 2$?*

(c) *Is average velocity positive or negative between $t = 3$ and $t = 7$?*

*** Relative Change**

When a quantity P changes from P_0 to P_1, we define

$$\text{Relative change in } P = \frac{P_1 - P_0}{P_0}.$$

The relative change is a number, without units. It is often expressed as a percentage.

Example 10 *On Black Monday, October 28, 1929, the stock market on Wall Street crashed. The Dow Jones average dropped from 298.94 to 260.64 in one day. What was the relative change in the index?*

1.4 Applications of Functions to Economics

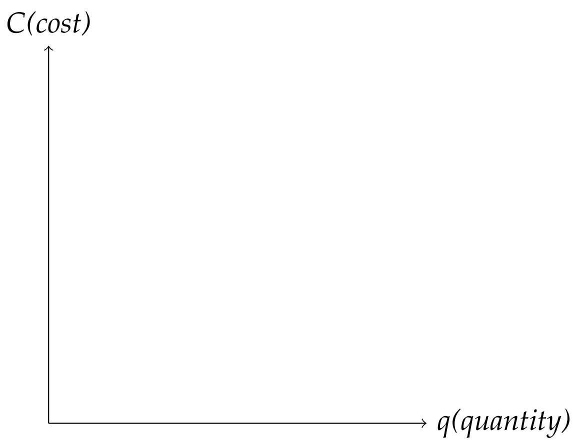
*** The Cost Function**

The **cost function**, $C(q)$, gives the total cost of producing a **quantity** q of some good.

$$\textbf{The total costs} = \textbf{Fixed Costs} + \textbf{Variable Costs},$$

where **Fixed Costs** are incurred even if nothing is produced and **Variable Costs** depend on how many units are produced.
If $C(q)$ is a linear cost function,
Fixed costs are represented by the vertical intercept.
Marginal cost is represented by the slope.

Example 1 *A company produces and sells shirts. The fixed costs are* \$7000 *and the variable costs are* \$5 *per shirt. Find a formula for the cost function* $C(q)$ *as a function of the quantity of shirts,* q*. And graph the cost function.*



*** The Revenue Function**

The **revenue function**, $R(q)$, gives the total revenue received from a firm from selling a **quantity**, q, of some good. If the good sells for a price of p per unit, then

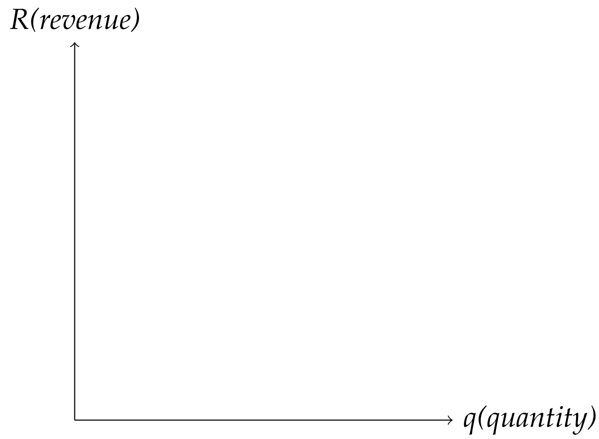
$$\textbf{Revenue} = \textbf{Price} * \textbf{Quantity},$$

which is exactly the same as

$$R = pq.$$

If the price does not depend on the quantity sold, so p is a constant, the graph of revenue as a function of q is a line through the origin, with slope equal to the price p.
The **marginal revenue** is also represented by the slope.

Example 2 *In Example 1, if the shirts are sold for* \$12 *each, find a formula for the revenue function* $R(q)$ *as a function of the quantity of shirts,* q*. And graph the revenue function.*



*** The Profit Function**

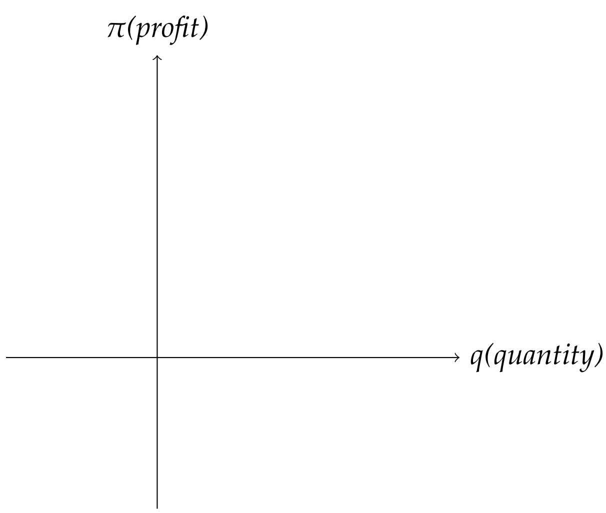
$$\textbf{Profit} = \textbf{Revenue} - \textbf{Cost}.$$

Let π denote the profit, then

$$\pi = R - C.$$

The **break-even** point is the point where the profit is zero, or equivalently, revenue equals cost.
If the profit function is a linear function, then the **marginal profit** is represented by the slope.

Example 3 *In Example 1 and Example 2, find a formula for the profit function* $\pi(q)$ *as a function of the quantity of shirts,* q*. Graph it and mark the break-even point.*



Example 4 *The following figure shows cost and revenue for a company.*

(a) Find the fixed costs and the marginal cost for the cost function $C(q)$.

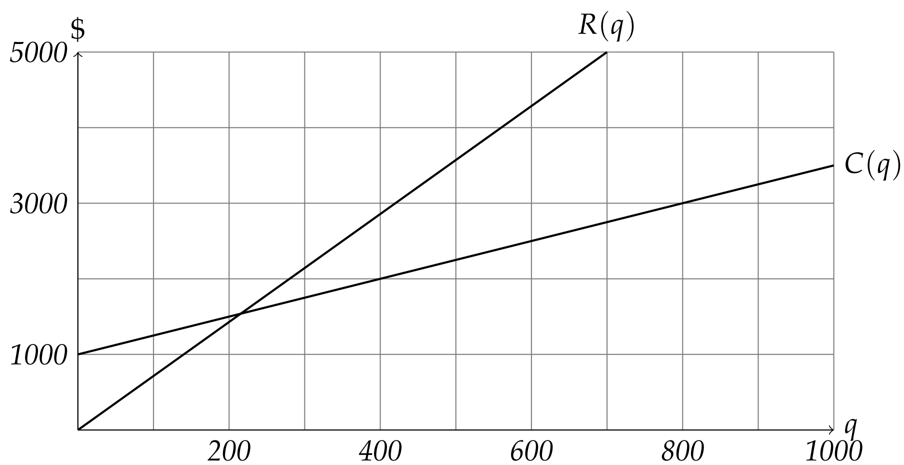
(b) Find a formula for $C(q)$.

(c) What is the marginal revenue?

(d) Find a formula for $R(q)$.

(e) Approximately what quantity does this company have to produce to make a profit?

(f) Estimate the profit generated by 500 units.



Example 5 *The following table shows a company's estimates of cost and revenue for a product.*

q	0	10	20	30	40
$C(q)$	500	600	700	800	900
$R(q)$	0	250	500	750	1000

(a) What are the fixed costs and the marginal cost?

(b) What price does the company charge for its products?

(c) Find a formula for $C(q)$ and $R(q)$.

(d) Find the break-even quantity.

Example 6 *A company that makes Adirondack chairs has fixed costs of* \$5000 *and variable costs of* \$30 *per chair. The company sells the chairs for* \$50 *each.*

(a) Find a formula for the cost function.

(b) Find a formula for the revenue function.

(c) Find the break-even point.

1.5 Exponential Functions

*** The General Exponential Function**

We say that P is an **exponential function** of t with base a if

$$P = P_0 a^t,$$

where P_0 is the initial quantity (when $t = 0$) and a is the factor by which P changes when t increases by 1. If $a > 1$, we have **exponential growth**; if $0 < a < 1$, we have **exponential decay**. The factor a is given by

$$a = 1 + r,$$

where r is the decimal representation of the percent rate of change; r may be positive (for growth) or negative (for decay).

Example 1 *World population is approximated by $P = 6.4(1.0126)^t$, with P in billions and t in years since 2004.*

(a) What is the yearly percent rate of growth of the world population?

(b) What was the world population in 2004? What does this model predict for the world population in 2012?

*** Comparison Between Linear and Exponential Functions**

A **linear** function has a constant rate of change.
An **exponential** function has a constant percent, or relative, rate of change.

Example 2 *The annual net sales for a chocolate company in 2008 was 5.1 billion dollars. In each of the following cases, write a formula for the annual net sales, S, of this company as a function of t, where t represents the number of years after 2008.*

(a) The annual net sales increases by 1.2 billion dollars per year.

(b) The annual net sales decreases by 0.4 billion dollars per year.

(c) *The annual net sales increases by* 4.3% *per year.*

(d) *The annual net sales decreases by* 1% *per year.*

*** Recognizing Data from an Exponential Function**

Values of t and P in a table could come from an exponential function $P = P_0a^t$ if ratios of P values are constant for equally spaced t values.

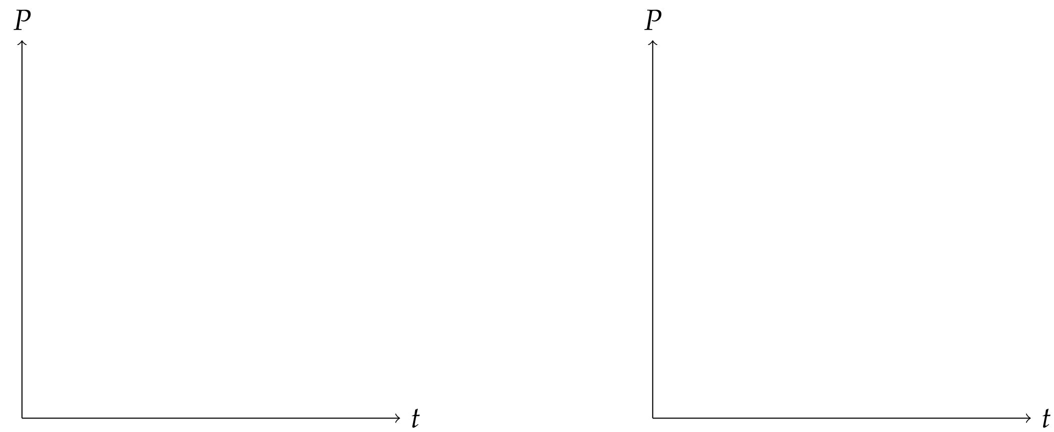
Example 3 *Which of the following functions in the following table could be linear, exponential, or neither? Find formulas for those functions.*

x	-2	-1	0	1	2
$f(x)$	500	600	700	800	900
$g(x)$	14	20	24	29	35
$h(x)$	16	24	36	54	81

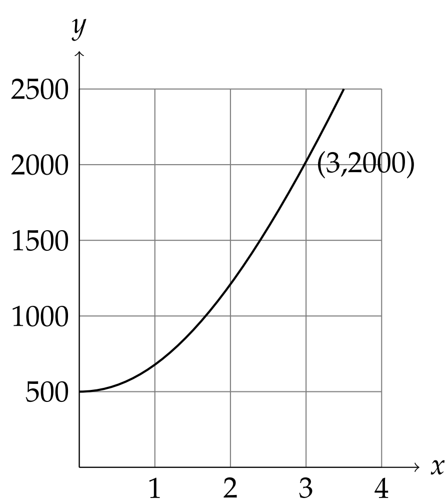
*** The Families of Exponential Functions and Number e**

The formula $P = P_0 a^t$ gives a family of exponential functions with parameters P_0 (the initial quantity) and a (the base).
If $a > 1$, then the function is increasing.
If $0 < a < 1$, then the functions is decreasing.
The larger a is, the faster the function grows; the closer a is to 0, the faster the functions decays.

The most commonly used base is the number $e = 2.71828...$, which is called the natural base.



Example 4 *Give a possible formula for the function which is represented by the following graph.*



1.6 The Natural Logarithm

*** Definition and Properties of the Natural Logarithm**

The **natural logarithm** of x, written $\ln x$, is the power of e needed to get x. In other words,

$$\ln x = c \qquad \text{means} \qquad e^c = x.$$

The natural logarithm is sometimes written $\log_e^x$.
$\ln x$ is not defined if x is negative or 0.

Properties of the Natural logarithm

$$\begin{aligned}
\ln(AB) &= \ln A + \ln B \qquad \text{(Product Rule)} \\
\ln\left(\frac{A}{B}\right) &= \ln A - \ln B \qquad \text{(Quotient Rule)} \\
\ln(A^p) &= p \ln A \qquad \text{(Power Rule)} \\
\ln e^x &= x \\
e^{\ln x} &= x
\end{aligned}$$

In addition, $\ln 1 = 0$ and $\ln e = 1$.

*** Solving Equations Using Logarithms**

Example 1 *Solve* $130 = 2^t$ *for* t *using natural logarithms.*

Example 2 *Solve* $100 = 25\,(1.5)^t$ *for* t *using natural logarithms.*

Example 3 *Solve* $5 = 2\,e^t$ *for* t *using natural logarithms.*

Example 4 *Solve* $5e^{3t} = 8e^{2t}$ *for* t *using natural logarithms.*

Example 5 *Solve* $7 \cdot 3^t = 5 \cdot 2^t$ *for* t *using natural logarithms.*

*** Exponential Functions with Base** e

Writing $a = e^k$, so $k = \ln a$, any exponential function can be written in two forms

$$P = P_0 a^t \qquad \text{or} \qquad P = P_0 e^{kt}.$$

If $a > 1$, we have exponential growth; if $0 < a < 1$, we have exponential decay.
If $k > 0$, we have exponential growth; if $k < 0$, we have exponential decay.
k is called the continuous growth or decay rate.

Example 6 *A town's population is 2000 and growing at 5% a year.*

(a) Find a formula for the population at time t years from now assuming that 5% per year is an annal rate.

(b) Find a formula for the population at time t years from now assuming that 5% per year is a continuous annual rate.

Example 7 *(a) Convert the function* $P = 20\,e^{-0.5t}$ *to the form* $P = P_0 a^t$.

(b) Convert the function $P = P_0\,e^{0.2t}$ *to the form* $P = P_0 a^t$.

(c) Convert the function $P = 10\,(1.7)^t$ *to the form* $P = P_0 e^{kt}$.

(d) Convert the function $P = 4\,(0.55)^t$ *to the form* $P = P_0 e^{kt}$.

Which of the functions above represents exponential growth and which represents exponential decay?

1.7 Exponential Growth and Decay

Many quantities in nature change according to an exponential growth or decay function of the form

$$P = P_0 e^{kt},$$

where P_0 is the initial quantity and k is the continuous growth or decay rate.

Example 1 *In 1990, the population of Africa was 643 million and by 2000 it had grown to 819 million.*

(a) Assuming the population increases exponentially at a continuous rate, find a formula for the population of Africa as a function of time t in years since 1990.

(b) By which year will Africa's population reach 2000 million?

*** Doubling Time and Half-Life**

The **doubling time** of an exponentially increasing quantity is the time required for the quantity to double.
The **half-life** of an exponentially decaying quantity is the time required for the quantity to be reduced by a factor of one half.

Example 2 *Find the doubling time of a quantity that is exponentially increasing by an annual rate of* 10% *per year.*

Example 3 *The quantity of ozone, Q, is decaying exponentially at a continuous rate of* 0.25% *per year. What is the half-life of ozone?*

Example 4 *Strontium-90 is a waste product from nuclear reactors, which decays exponentially The half-life of strontium-90 is 29 years. Estimate the percent of original strontium-90 remaining after 100 years?*

*** Financial Applications: Compound Interest**

An amount P_0 is deposited in an account paying interest at a rate of r per year. Here r is the decimal representation of the percentage. Let P_0 be the initial deposit. Let P be the balance in the account after t years.
If interest is **compounded annually**, then $P = P_0\,(1+r)^t$.
If interest is **compounded continuously**, then $P = P_0\,e^{rt}$, where e is the natural base.

Example 5 *If you deposit* \$3000 *in an account earning interest at an* 8% *annual rate. How much is in the account after 10 years if the interest is compounded*

(a) Annually?

(b) Continuously?

Example 6 *If* 10,000 *is deposited in an account paying* 10% *interest per year, compounded continuously, how long will it take for the balance to reach* 25,000*?*

1.8 New Functions From Old

*** Composite Functions**

Suppose that $f(x)$ and $g(x)$ are functions, the function $f(g(x))$ is called a **composite function**, in which there is an **inside function** g and an **outside function** f.

Example 1 *For* $f(x) = -2x^2 + x + 5$, *find and simplify:*

(a) $f(x+h)$

(b) $f(x+h) - f(x)$

Example 2 *If* $f(x) = x^2 - 2$, *find and simplify:*

(a) $f(t-1)$

(b) $f(t^2-1)$

(c) $f(3)$

(d) $4f(t)$

(e) $[f(t)]^2 - 2$

Example 3 *If* $f(x) = 2x^2$ *and* $g(x) = x + 3$, *find the following:*

(a) $f(g(1))$, $g(f(1))$, $f(1) \cdot g(1)$ *and* $f(1) + g(1)$

(b) $f(g(x))$

(c) $g(f(x))$

(d) $f(x) \cdot g(x)$

(e) $f(f(x))$

Example 4 *Using the following table, find* $g(f(0))$, $f(g(0))$, $g(f(1))$, *and* $f(g(1))$.

x	0	1	2	3
$f(x)$	3	1	-1	-3
$g(x)$	0	2	4	6

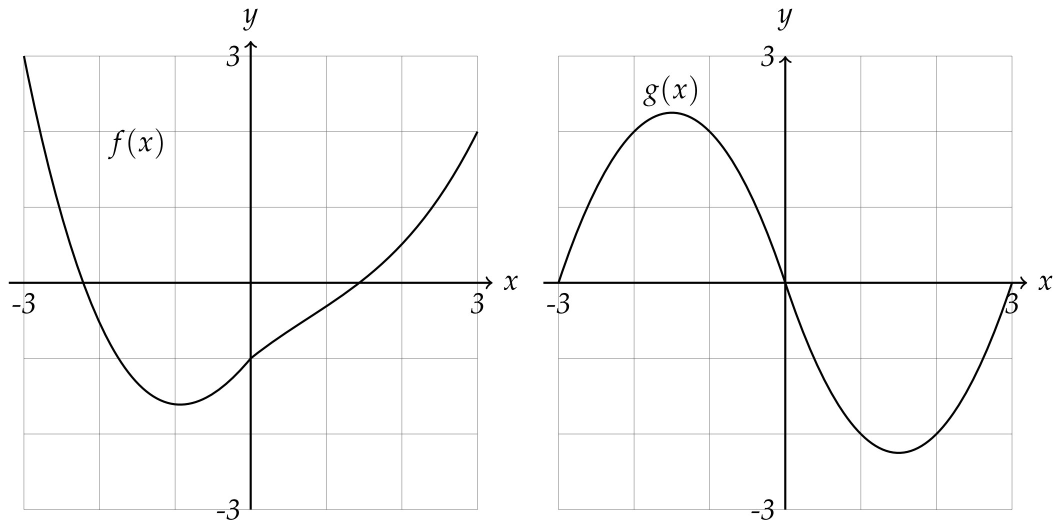
Example 5 *Use the variable u for the inside function to express each of the following as a composite function:*

(a) $C = 5\ln(x^2 + 1)$

(b) $P = 64\, e^{-0.12t}$

(c) $y = \sqrt{x^2 + 4}$

Example 6 *Given the graph of $f(x)$ and $g(x)$ in the below. Estimate $f(g(1))$, $g(f(2))$ and $f(f(1))$.*



*** Stretches of Graphs**

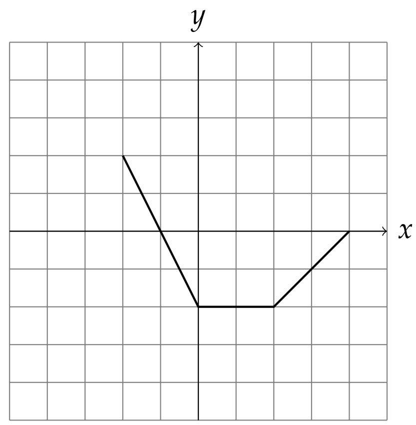
> Multiplying a function by a constant, c, stretches the graph vertically (if $c > 1$) or shrinks the graph vertically (if $0 < c < 1$). A negative sign (if $c < 0$) reflects the graph about the x-axis, in addition to shrinking or stretching.

*** Shifted Graphs**

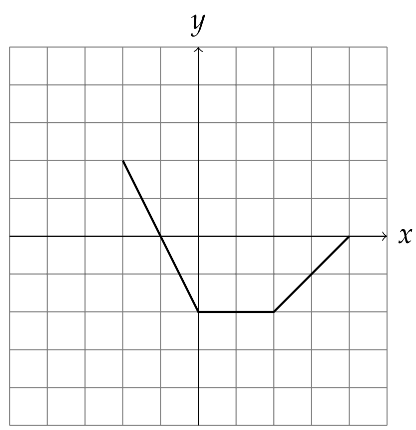
> Assume that $k > 0$.
> The graph of $y = f(x) + k$ is the graph of $y = f(x)$ moved up k units.
> The graph of $y = f(x) - k$ is the graph of $y = f(x)$ moved down k units.
> The graph of $y = f(x + k)$ is the graph of $y = f(x)$ moved to the left k units.
> The graph of $y = f(x - k)$ is the graph of $y = f(x)$ moved to the right k units.

Example 7 *Use the following graph of the function $f(x)$ to sketch the graph of the following functions:*

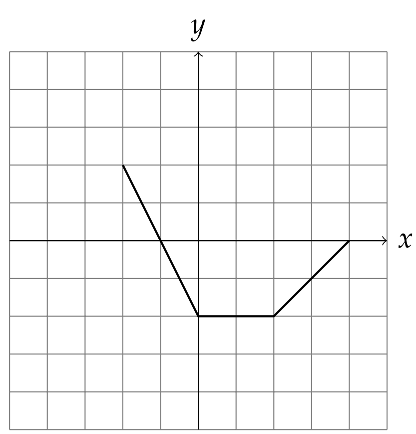
(a) $g(x) = f(x) + 3$



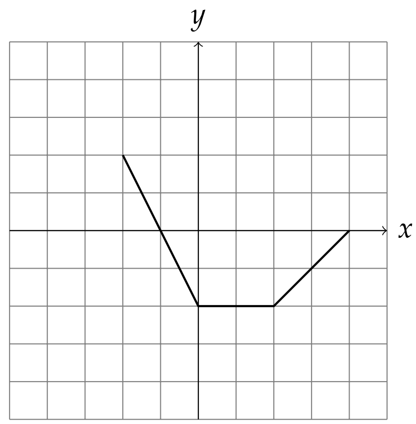
(b) $v(x) = f(x) - 2$



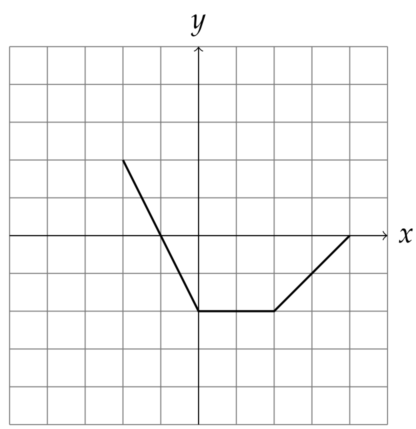
(c) $h(x) = f(x+2)$



(d) $u(x) = f(x-1)$



(e) $w(x) = -f(x) + 2$



Example 8 *(a) Write an equation for a graph obtained by vertically stretching the graph of $y = x^2$ by a factor of 2, followed by a vertical upward shift of 1 unit. Sketch it.*

(b) What is the equation if the order of the transformations (stretching and shifting) in part (a) is interchanged? sketch it.

1.9 Proportional and Power Functions

*** Proportionality**

We say y is (directly) **proportional** to x if there is a nonzero constant k such that

$$y = kx.$$

This k is called the constant of proportionality.
We say y is **inversely proportional** to x if there is a nonzero constant k such that

$$y = \frac{k}{x}.$$

Or equivalently, if the product of x and y equals a constant k, then y is **inversely proportional** to x.

Example 1 *The blood mass of a mammal is proportional to its body mass.*

(a) Write a formula for blood mass, B, as a function of body mass, M.

(b) A rhinoceros with body mass 3000 kilograms has blood mass of 150 kilograms. Use this information to find the constant of proportionality.

(c) Estimate the blood mass of a human with body mass 70 kilograms.

Example 2 *The number of animal species of a certain body length, N, is inversely proportional to the square of the body length, L. Write a formula for N as a function of L. Are there more species at large lengths or at small lengths?*

Example 3 *Use the following tables to determine whether $f(x)$ and $g(x)$ are proportional or inversely proportional to x? If so, find the constant of proportionality and write a formula for the corresponding function.*

(a)

x	-3	0	6	9	12
$f(x)$	60	0	-120	-180	-240

(b)

x	-2	2	6	10	14
$g(x)$	105	-105	-35	-21	-15

*** Power Functions**

We say $Q(x)$ is a **power function** of x if $Q(x)$ is proportional to a constant power of x. If k is the constant of proportionality, and if p is the power, then

$$Q(x) = k \cdot x^p.$$

Example 4 *Which of the following are power functions? For those which are, write the function in the form $y = kx^p$, and give the coefficient k and the exponent p.*

(a) $y = \dfrac{10}{x^4}$

(b) $y = 6 \cdot 4^x$

(c) $y = 9\sqrt{x}$

(d) $y = (2x^3)^2$

(e) $y = x^8 + 1$

(f) $y = \dfrac{5}{3\sqrt{x}}$

(g) $y = \dfrac{x}{9}$

*** Graphs of Power Functions**

*** Quadratic Functions and Polynomials**

Sums of power functions with nonnegative integer exponents are called **polynomials**, which are functions of the form

$$y = p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0.$$

Here, n is a nonnegative integer, called the **degree** of the polynomial, and a_n is a nonzero number called the **leading coefficient**. We call $a_n x^n$ the **leading term**.
If $n = 2$, the polynomial is called **quadratic**.

Example 5 *Which of the following functions are polynomial functions? For those which are, give the degree n and the leading coefficient a_n.*

(a) $3x^{-2} + 1$

(b) $7x^{10} + x^2$

(c) $2^x + 3$

(d) $2\sqrt{x} + x - 1$

(e) $8x^6 + x^2 - 4x + 2 - 8x^6$