SOLVE THE FOLLOWING PROBLEMS. **You must show your work!**
Points may be withdrawn for the answers given without substantiation!

1. (10 pts) Evaluate the limit

\[
\lim_{x \to 0} \frac{\sin(x)}{3x + 2\tan(x)} = \frac{\sin(x)}{3x} \cdot \frac{1}{1 + \frac{2\tan(x)}{3x}} = \frac{\sin(x)}{3x} \cdot \frac{1}{1 + \frac{2\sin(x)}{3x} \cdot \frac{1}{\cos(x)}} = \frac{1}{5} \quad \text{as } x \to 0
\]

2. Find the derivatives of the following functions:
   a. (10 pts)

\[
f(s) = (4s - s^2)^{100}
\]

\[
f'(s) = 100(4s - s^2)^{99} \cdot (4 - 2s) = 200(4s - s^2)^{99}(2 - s)
\]

b. (10 pts)

\[
f(t) = \tan(5 - \sin(2t))
\]

\[
f'(t) = \sec^2(5 - \sin(2t)) \cdot (-2 \cos(2t))
\]

\[
= -2 \sec^2(5 - \sin(2t)) \cdot \cos(2t)
\]

c. (10 pts)

\[
g(x) = \frac{\tan(3x)}{x^2 + 1}
\]

\[
g'(x) = \frac{3(\sec^2(3x))(x^2 + 1) - 2x \cdot \tan(3x)}{(x^2 + 1)^2}
\]

\[
= \frac{3(x^2 + 1) \sec^2(3x) - 2x \cdot \tan(3x)}{(x^2 + 1)^2}.
\]
3. (10 pts) Find the 25th derivative of the function

\[ y = \cos(x) + 73x^{16} - 108x^5 + 7. \]

\[
\begin{align*}
(\cos x)' &= -\sin x, & (\cos x)'' &= -\cos x, & (\cos x)^{(3)} &= \sin x \\
(\cos x)^{(4)} &= \cos x, & (\cos x)^{(5)} &= -\sin x
\end{align*}
\]

\[
\therefore \quad y^{(5)} = (\cos x)^{(5)} = (\cos x)^{(6\times4+1)} = (\cos x)' = -\sin x
\]

4. (10 pts) Let \( x + \tan(xy) = 0 \), find \( \frac{dy}{dx} \).

\[
\frac{d}{dx} \left( x + \tan(xy) \right) = 0 \\
1 + \sec^2(xy) \cdot \frac{dy}{dx} = 0 \\
1 + \sec^2(xy) \cdot \left( y + x \frac{dy}{dx} \right) = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{1 + y\sec^2(xy)}{\sec^2(xy)}
\]

5. (10 pts) When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50 cm?

Let the radius of the plate be \( r \) cm & the area of the plate be \( A \) cm\(^2\).

Then, \( A = \pi r^2 \)

Take the derivative w.r.t. time at both sides of equation.

\[
\frac{dA}{dt} = 2\pi r \frac{dr}{dt}
\]

Since \( \frac{dr}{dt} = 0.01 \text{ cm/min} \), when \( r = 50 \text{ cm} \), \( \frac{dA}{dt} = 1\pi \text{ cm}^2/\text{min} \).
6. Let 
\[ f(x) = (1 + x)^k, \]

a (10 pts) Find the linearization of \( f(x) \) at \( x = 0 \).
\[
\frac{f'(x)}{f(x)} = k(1+x)^{k-1}.
\]
\[
\text{At } x = 0:
\]
\[
L(x) = f(0) + f'(0) \cdot (x - 0) = 1 + kx.
\]

b (5 pts) Use the linearization that you find in [a] to estimate \((1.0002)^{50}\).
\[
(1 + 0.0002)^{50} = (1.0002)^{50} \approx L(0.0002) = L(0) + 50 \cdot 0.0002 = 1 + 0.1 = 1.1
\]

7. Let \( f(x) = x^{2/3} \) on \([-1, 2]\).

a (5 pts) Find the critical number of \( f(x) \) on \([-1, 2]\).
\[
f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}
\]
\[
f'(x) \text{ does not exist at } x = 0.
\]
\[
\Rightarrow \text{ The critical number of } f(x) \text{ on } [-1, 2] \text{ is } x = 0
\]

b (10 pts) Find the global (absolute) maximum and minimum values of \( f(x) \) on \([-1, 2]\).

Compare the values of \( f(x) \) at the critical number and two end points.

We know that \( f(x) \) has global max \( \sqrt[3]{4} \) at \( x = 2 \) and global min 0 at \( x = 0 \).