1. (24 pts) Given \( A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{bmatrix} \), verify that \( A \) is nonsingular, and then compute \( A^{-1} \) by two distinct methods.

2. (16 pts) The matrix \( A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ -1 & 1 & -2 & -1 \\ 1 & 2 & -1 & 1 \end{bmatrix} \) is row equivalent to \( R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \).

Use this information to solve the equation \( Ax = 0 \).

3. (16 pts) If \( \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \), find \( A^{-1} \).

4. (20 pts) True or false. If your answer is false, please explain. All matrices are \((n \times n)\).

   (a) If the row echelon form of \( A \) involves free variables, then the system \( Ax = b \) will have infinitely many solutions.

   (b) If \( \exists \, x \in \mathbb{R}^n, \, x \neq \theta \) such that \( Ax = 0 \), then \( \det(A) = 0 \).

   (c) \( (A - B)^2 = A^2 - 2AB + B^2 \)

   (d) If \( A \) and \( B \) are both nonsingular, then \( A \sim B \).

   (e) \( \det(AB) = \det(BA) \).

5. (12 pts) Given \( A = \begin{bmatrix} 1 & -2 & 3 \\ 3 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix} \).

   (a) Reduce \( A \) to upper triangular form using only type III row operations.

   (b) Is \( A \) nonsingular? Why?

6. (12 pts) (a) Let \( A, B \in \mathbb{R}^{m \times n} \). What does it mean to say \( A \sim B \) ?

   (b) Prove that if \( A \sim B \) and \( B \sim C \), then \( A \sim C \).