Explicit subspaces in Dvoretzky’s theorem

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Let \((X, \|\cdot\|)\) be a real normed space of dimension \(n \in \mathbb{N}\) with a basis \(\{e_i\}_{i=1}^n\) of universally bounded basis constant such that with respect to this basis, the norm is invariant under coordinate permutations. Let \(0 < \varepsilon < c\) (where \(c > 0\) is a universal constant) and consider any \(k \in \mathbb{N}\) with \(k \leq c (\log \varepsilon^{-1})^{-1} \log n\). We provide an explicit construction of a matrix that generates a \((1 + \varepsilon)\) embedding of \(\ell_2^k\) into \(X\).

A large body of literature exists in the case \(X = \ell_1^n\), however this is the first explicit construction of subspaces in Dvoretzky’s theorem that applies to a wide class of spaces, and the bound for \(k\) is optimal (in the general setting) in terms of \(n\) and \(\varepsilon\), up to a universal constant. Our result extends non-explicit results of Bourgain/Lindenstrauss and Tikhomirov on symmetric spaces.

In this talk we discuss the above result, as well as some of its limitations and directions for future research.