From Wade

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Non-book Exercises

1) For which $\alpha > 0$ is the function

$$f(x, y) = \begin{cases} \frac{x^2|y|^\alpha}{x^2 + |y|^2} & (x, y) \neq 0 \\ 0 & (x, y) = 0 \end{cases}$$

differentiable at zero?

Honor’s Problems

2) Define the space $C^1[a,b] = \{ f : [a, b] \mapsto \mathbb{R} \mid f \text{ and } f' \in C[a,b] \}$, and the norm

$$\|f\|_{1,1} = \int_a^b (|f(x)| + |f'(x)|) \, dx.$$

(a) Show there exists $M > 0$ such that for all $f \in C^1[a,b]$, $\|f\|_\infty \leq M\|f\|_{1,1}$.

*Hint: Use the Fundamental Theorem of Calculus.*

(b) Show that if $\{f_n\}_{n=1}^\infty$ is a sequence from $C^1$ and $f_n \rightarrow g$ in $\|\cdot\|_{1,1}$ then $f_n \rightarrow g$ point wise.

(c) Define $W^{1,1}$ to be the set of all sequences from $C^1$ which are Cauchy in the norm $\|\cdot\|_{1,1}$. Show that for any sequence $\{f_n\}$ from $C^1$ which is Cauchy in $\|\cdot\|_{1,1}$ there is a $g \in C[a,b]$ such that

$$\|f_n - g\|_1 \rightarrow 0.$$

For this reason we say that

$$W^{1,1} \subset C[a,b].$$
3) (a) Let \( f \in L^1[a, b] \) and \( g \in W^{1,1}[a, b] \). Show that the product \( fg \in L^1[a, b] \). That is, if \( f \) is represented by the \( \| \circ \|_1 \) Cauchy sequence \( \{f_n\} \subset C[a, b] \) and \( g \) by the \( \| \circ \|_{1,1} \) Cauchy sequence \( \{g_n\} \subset C^1[a, b] \), then the sequence \( \{h_n\} \) where \( h_n = f_n g_n \) is contained in \( C[a, b] \) and is Cauchy in \( \| \circ \|_1 \).

(b) In part (a), show that if \( g \) is merely in \( L^1[a, b] \), then the product \( fg \) may not be in \( L^1[a, b] \). That is find two sequences \( \{f_n\} \) and \( \{g_n\} \), both from \( C[a, b] \) and both Cauchy in \( \| \circ \|_1 \) such that the “product” \( \{f_n g_n\} \) is not Cauchy in \( \| \circ \|_1 \).