Non-Book Problems

1) Let $\Gamma$ be a curve in $\mathbb{R}^3$,
$$\Gamma = \left\{ \vec{\gamma}(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t)) \bigg| t \in [0, 1] \right\},$$
where $\vec{\gamma} \in C^2([0, 1], \mathbb{R}^3)$ satisfies $\|\vec{\gamma}'(t)\| = 1$ for all $t \in [0, 1]$. Suppose that $\vec{\phi}$ and $\vec{\psi}$ are $C^1([0, 1], \mathbb{R}^3)$ functions which satisfy $\|\vec{\phi}(t)\| = \|\vec{\psi}(t)\| = 1$.

Find conditions on $\vec{\phi}$ and $\vec{\psi}$ under which the map $F : \mathbb{R}^6 \mapsto \mathbb{R}^3$ given by
$$F(\vec{x}, \vec{t}, s_1, s_2) := \vec{\gamma}(t) + s_1 \vec{\phi}(t) + s_2 \vec{\psi}(t) - \vec{x},$$
can be used to solve for $t = t(\vec{x})$, $s_1 = s_1(\vec{x})$, and $s_2 = s_2(\vec{x})$ such that $F(\vec{x}, t(\vec{x}), s_1(\vec{x}), s_2(\vec{x})) = 0$, for $\vec{x}$ close enough to $\Gamma$.

2) Let $\Gamma$ be a smooth two-dimensional submanifold of $\mathbb{R}^3$, ie.
$$\Gamma = \left\{ \vec{\gamma}(\vec{t}) = (\gamma_1(\vec{t}), \gamma_2(\vec{t}), \gamma_3(\vec{t})) \bigg| \vec{t} = (t_1, t_2) \in [0, 1] \times [0, 1] \right\},$$
where the smooth function $\vec{\gamma}$ satisfies $\left| \frac{\partial \vec{\gamma}}{\partial t_1}(\vec{t}) \times \frac{\partial \vec{\gamma}}{\partial t_2}(\vec{t}) \right| = 1$. Let $\vec{\nu}(\vec{t})$ be the normal to $\Gamma$ at $\vec{\gamma}(t)$. Show that the map
$$F(\vec{x}, \vec{t}, s) := \vec{\gamma}(t) + s \vec{\nu}(t) - \vec{x},$$
has can be solved for $\vec{t} = \vec{t}(\vec{x})$ and $s = s(\vec{x})$ such that $F(\vec{x}, \vec{t}(\vec{x}), s(\vec{x})) = 0$ for all $\vec{x}$ close enough to $\Gamma$. 