SHOW ALL YOUR WORK. NO WORK MEANS NO CREDIT!

1. (5 points) Let \( f(x) = 2x^2 + x \) for all \( x \in [1, 3] \). Note that \( f \) is \underline{continuous on [1, 3]} and is \underline{differentiable on (1, 3)}. Thus by the \textbf{Mean Value Theorem}, there exists a number \( c \in (1, 3) \) such that \( f'(c) = \frac{f(3) - f(1)}{3 - 1} \).

Find such a number \( c \) (or numbers \( c \)).

\[
\begin{align*}
\text{First,} & \quad \frac{f(3) - f(1)}{3 - 1} = \frac{18 + 3 - (2 + 1)}{2} = \frac{18}{2} = 9 \\
& \quad f'(x) = 4x + 1 \\
& \quad f'(c) = 4c + 1 = 9 \\& \quad 4c = 8 \\& \quad c = 2 \in (1, 3).
\end{align*}
\]

2. (2+2+2+2+5 points) Let \( f(x) = x + \frac{4}{x} \).

(a) Find the domain of \( f \).

\[
(-\infty, 0) \cup (0, \infty)
\]

(b) List the critical point(s) of \( f \).

\[
f'(x) = 1 - \frac{4}{x^2} = \frac{x - 4}{x^2} = 0
\]

\[
\Rightarrow x = \pm 2 \quad \text{each} \quad +1
\]
(c) Find the interval of \( x \) for which \( f'(x) > 0 \).

\[
\begin{array}{cccc}
\text{Domain} \\
-2 & 0 & 2 \\
\hline
f(x) & + & - & + \\
\end{array}
\]

\((-\infty, -2) \cup (2, \infty)\)

(d) Find the interval of \( x \) for which \( f'(x) < 0 \).

\((-2, 0) \cup (0, 2)\)

(Note: 0 is not in the domain)

(e) At what points of \( x \), do the local maxima and minima occur?

By the first derivative test with the table in (c),

- \( f \) has a local max at \( x = -2 \)
- \( f \) has a local min at \( x = 2 \)

(f) Find the absolute maximum and minimum values of \( f \) on the interval \([1, 3]\) and specify the points of \( x \) at which they occur. (You can use part (b).)

The only critical pt of \( f \) in \((1, 3)\) is \( x = 2 \).

\begin{align*}
\text{Critical} & \quad f(2) = 2 + \frac{4}{2} = 4 \\
\text{end points} & \quad f(1) = 1 + \frac{4}{1} = 5 \\
& \quad f(3) = 3 + \frac{4}{3} = 4 + \frac{1}{3} \\
\end{align*}

\( f(2) \) is the abs. min. value of \( f \) on \([1, 3]\) which occurs at \( x = 2 \).

\( f(1) \) is the abs. max. value of \( f \) on \([1, 3]\) which occurs at \( x = 1 \).