1. (10+10+10 pts) Find the derivatives of the following functions.

(a) \( f(x) = (x^3 + x^2 + x + 1)^{2015} \)

\[
\frac{d}{dx} \left( x^3 + x^2 + x + 1 \right)^{2015} = 2015 \cdot \frac{d}{dx} \left( x^3 + x^2 + x + 1 \right) \cdot \left( x^3 + x^2 + x + 1 \right)^{2014} \\
= 2015 \cdot \left( 3x^2 + 2x + 1 \right) \cdot \left( x^3 + x^2 + x + 1 \right)^{2014}
\]

(b) \( g(s) = \frac{\sin s + \sec s}{\sqrt{s} + s^{-\frac{3}{2}}} \)

\[
g'(s) = \frac{\left( \cos s + \sec s \tan s \right) \left( \sqrt{s} + s^{-\frac{3}{2}} \right) - \left( \frac{1}{2} s^{-\frac{1}{2}} - \frac{1}{3} s^{-\frac{5}{2}} \right) \left( \sin s + \sec s \right)}{\left( \sqrt{s} + s^{-\frac{3}{2}} \right)^2}
\]

(c) \( h(t) = (\tan t + t^5)(t^2 - 3t + 1) \)

\[
h'(t) = \left( \sec^2 t + 5t^4 \right) (t^2 - 3t + 1) + (\tan t + t^5) (2t - 3)
\]

2. (10 pts) Use the limit definition of a derivative to find the derivative of the following function

\( f(x) = 3\sqrt{x + 1} \)

\[
f'(x) = \lim_{h \to 0} \frac{3\sqrt{x+h+1} - 3\sqrt{x+1}}{h} = 3 \cdot \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}
\]

\[
= 3 \cdot \frac{h}{2(x+1)} = \frac{3}{2\sqrt{x+1}}
\]
3. (10 pts) Find the following limit:
\[
\lim_{\theta \to 0} \frac{2 \sin^2 \theta}{\theta^2 \cos \theta}
\]
\[
= \lim_{\theta \to 0} \frac{2 \sin \theta \cdot \sin \theta}{\theta \cos \theta} \cdot \frac{1}{1}
\]
\[
= \frac{2 \cdot 1 \cdot 1}{1} = 2.
\]
(since \( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \))

4. (10 pts) For a function \( y = y(x) \) implicitly defined by \( xy^2 + y \cos x + x^4 = 132 \), find \( \frac{dy}{dx} \)

Take \( \frac{dy}{dx} \) on both sides,
\[
\Rightarrow \frac{y^2}{dx} + 2xy \cdot \frac{dy}{dx} + \frac{dy}{dx} \cos x - y \sin x + 4x^3 = 0
\]
\[
\Rightarrow \frac{dy}{dx} (2xy + \cos x) = -y^2 + y \sin x - 4x^3
\]
\[
\Rightarrow \frac{dy}{dx} = \frac{-y^2 + y \sin x - 4x^3}{2xy + \cos x}
\]

5. (10+5 pts) A particle is moving on a straight line according to the law of motion
\( s(t) = t^2 - 4t + 20 \) (\( s(t) \) in feet, \( t \) in seconds).

(a) What is the initial velocity (that is, velocity when \( t = 0 \))? Is the particle moving in the positive or negative direction at this moment?

\[ v(t) = s'(t) = 2t - 4 \]
\[ v(0) = -4 \text{ (ft/s)} \]
\[ \text{Negative direction} \]

(b) When does the particle change its direction of movement for \( t > 0 \)?

\[ v(t) = 2(t - 2) \]
\[ v(t) > 0 \text{ for } t > 2 \]
\[ v(t) = 0 \text{ at } t = 2 \]
\[ v(t) < 0 \text{ for } 0 < t < 2 \]
So at \( t = 2 \), the particle changed its direction from negative to positive direction.
6. (5+5 pts) Let \( f(x) = \sqrt{x} \).

(a) Find the linearization \( L(x) \) of \( f(x) \) at \( x = 8 \).

\[
L(x) = \frac{f(0) + f'(0)(x-8)}{2} + 1
\]

\[
f'(x) = \frac{1}{2} x^{-\frac{1}{2}}
\]

\[
f'(8) = \frac{1}{3} \cdot \frac{1}{8^{\frac{1}{2}}} = \frac{1}{12}
\]

\[
= \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}
\]

+2

(b) Use part (a) to estimate \( \sqrt{9} \). Write your answer as a single fraction of integers.

\[
\sqrt{9} = \frac{2 + \frac{1}{12}(9-8)}{2}
\]

\[
= \frac{2 + \frac{1}{12}}{2} = \frac{25}{12}
\]

\( = 2.083333 \ldots \)

\[
\text{NOTE: } \sqrt{9} = 2.080083823 \ldots
\]

7. (15 pts) There is a spherical balloon whose radius is increasing at the rate of 2 in/sec. What are the rates of change of the volume and surface area for the balloon when the radius is 5 inches. (Note that the volume and surface area of the sphere with radius \( r \) are \( V = \frac{4}{3} \pi r^3 \) and \( S = 4\pi r^2 \), respectively.)

\[
\begin{aligned}
& \text{Given } \frac{dr}{dt} = 2, \\
& \text{to find } \frac{dV}{dt}, \frac{dS}{dt} \text{ when } r = 5.
\end{aligned}
\]

\[
V = \frac{4}{3} \pi r^3, \quad S = 4\pi r^2
\]

\[
\Rightarrow \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} = 8\pi r \cdot \frac{dr}{dt}
\]

\[
\begin{aligned}
& \text{when } r = 5, \quad \frac{dV}{dt} = 4\pi \cdot 5 \cdot 2 = 8\pi \cdot 5 \cdot 2
\end{aligned}
\]

\[
\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} = 8\pi \cdot 5 \cdot 2
\]

\[
= 200\pi \left( \frac{m^3}{\text{sec}} \right) = 80\pi \left( \frac{m^3}{\text{sec}} \right)
\]

+3

\[
\frac{dV}{dt} = 80\pi \left( \frac{m^3}{\text{sec}} \right)
\]

+3