1. (10+10+10 pts) Find the following limits.

(a) \( \lim_{x \to 2^-} \frac{5}{(x-2)^2} = \infty \)  

(b) \( \lim_{x \to 1} \frac{x+1}{x^2-x+1} = \frac{1+1}{1^2-1+1} = 2 \)

(c) \( \lim_{x \to 1} \frac{x-1}{x^2+x-2} = \lim_{x \to 1} \frac{1}{(x-1)(x+2)} = \lim_{x \to 1} \frac{1}{x+2} = \frac{1}{3} \)

2. (10 pts) Find the vertical asymptote(s) of the following function:

\[ f(x) = \frac{2(x+2)}{x(x-1)} \]

\[ \begin{align*}
\lim_{x \to 0^-} & \frac{2(x+2)}{x(x-1)} = +\infty \\
\lim_{x \to 1^-} & \frac{2(x+2)}{x(x-1)} = -\infty \\
\lim_{x \to 1^+} & \frac{2(x+2)}{x(x-1)} = +\infty \\
\lim_{x \to 0^-} & \frac{2(x+2)}{x(x-1)} = -\infty \\
\end{align*} \]
3. (5+5+5 pts) Find the following limits.

\[
\lim_{x \to -2} \frac{x - 2}{x - 2} = \frac{-1}{0} + 5 \quad \lim_{x \to -2} \frac{x - 2}{(x - 2)} = -1
\]

\[
\lim_{x \to 2^+} \frac{x - 2}{x - 2} = \frac{1}{0} + 5 \quad \lim_{x \to 2^+} \frac{x - 2}{x - 2} = 1
\]

\[
\lim_{x \to 2} \frac{x - 2}{|x - 2|} = \text{DNE} + 5
\]

4. (10 pts) Knowing that \(9x - 10 \leq g(x) \leq 2x^2 + x - 2\) for all \(x\), find \(\lim_{x \to 2} g(x)\) and specify the name of the theorem you are using.

\[
\lim_{x \to 2} (9x - 10) = 8 \quad + 3
\]

\[
\lim_{x \to 2} (2x^2 + x - 2) = 8 + 2 - 2 = 8 \quad + 3
\]

\[
\lim_{x \to 2} g(x) = 8 \quad + 2, \text{ by the squeeze theorem}
\]

5. (15 pts) Use the Intermediate Value Theorem to show that \(g(x) = \frac{1}{x} - 3x^4 + 6\) has a zero in the interval \((0, \infty)\).

\[
g(1) = 1 - 3 + 6 = 4 > 0 \quad + 4
\]

\[
g(2) = \frac{1}{2} - 48 + 6 = \frac{1}{2} - 42 < 0 \quad + 4
\]

\[
g \quad \text{is continuous on} \quad (0, \infty), \text{ i.e., on} \quad [1, 2] \quad + 4
\]

By the Intermediate Value Theorem, \([\text{there is a number} \quad c \in (1, 2) \text{ with} \quad g(c)] \quad + 3\)
6. (9+11 pts) Let

\[ f(x) = \begin{cases} \frac{x-4}{x^2-16} & \text{if } x \neq \pm 4, \\ 3 & \text{if } x = 4, \\ 0 & \text{if } x = -4. \end{cases} \]

(a) Find the interval (or the union of intervals) in which the function \( f(x) \) is continuous.

\[ f(x) = \frac{x-4}{x^2-16} = \frac{x-4}{(x+4)(x-4)} = \frac{1}{x+4} \quad \text{for } x \neq \pm 4. \]

\[ \lim_{x \to 4} f(x) = \frac{1}{8}, \quad \lim_{x \to -4} f(x) \text{ DNE}, \quad \lim_{x \to -4} 3 = f(4), \quad \lim_{x \to 4} 3 = f(4), \quad \lim_{x \to -4^+} \frac{1}{x+4} = +\infty. \]

\[ 8, \frac{1}{8}, f \text{ is continuous on } (-\infty, -4) U (-4, 4) U (4, \infty). \]

(b) Is/are there a removable discontinuity/removable discontinuities? If yes, state the value(s) of \( x = a \) at which it occurs and redefine \( f(a) \) to remove the discontinuity.

By the above, \( \lim_{x \to 4} f(x) = \frac{1}{8} \), the discontinuity of \( f \) at \( x = 4 \) is removable.

Redefine \( f(4) = \frac{1}{8} \). Then \( f \) becomes continuous at \( x = 4 \).