Overview/Scope/Prerequisites: The overarching goal of the course is to give a broad introduction to knot theory and the many areas of low-dimensional topology with which it interacts. It will take participants from the beginnings of the subject to the forefront of exciting recent developments. Emphasize will be placed on open problems, conjectures and current directions of research, and the course should provide stimulation for further independent reading and research. The only formal prerequisite will be to have taken 868-869 or the equivalent.

Description: Topics will be selected from: Knot diagrams and their invariants, Braid groups, Seifert Forms, Cyclic covers of knot complements and their invariants, Jones type polynomial invariants, quantum invariants, surfaces in knot complements, hyperbolic knots, relations between quantum invariants and geometric structures of knots and concordance groups.

Some textbooks:

- Cromwell: Knots and links by Peter Cromwell
- Murasugi: Knot theory by Kunio Murasugi
- Prasolov-Sossinsky: Knots, links, braids and 3-manifolds by V.V.Prasolov and A.B.Sossinsky
- Rolfsen: Knots and links by Dale Rolfsen

Outline and References:

Note: The text in Red shows hyperlinks. Click for more details.

1. Knots and links: Topics selected from: Definitions, diagrams, Reidemeister moves, wild knots, connect sum, unknotting, linking number, Dowker-Thistlethwaite code, satellite knots, famous families of knots, [historical overview of knot theory].
   Reading: Lickorish ch 1, Cromwell ch 1-4, Prasolov-Sossinsky ch 1.

2. Braids and tangles: Topics selected from: Braid group, Alexander and Markov theorems, braid index. Rational tangles, rational links, 2-bridge links, plats.
   Reading: Prasolov-Sossinsky ch 3, Cromwell ch 8, 10.1, 10.4.

Reading: Lickorish ch 2,8, Cromwell ch 5-6, Murasugi ch 5, 6.4.


Reading: Lickorish ch 6,7,11, Cromwell ch 7, Rolfsen ch 6-8, Murasugi ch 6.


Reading: Lickorish ch 3,5, Prasolov-Sossinsky ch 2.3, Cromwell ch 9.


Reading:
- Bollobas Modern Graph Theory ch 10,
- Jones Expo 1
- Jones Expo 2
- The Jones polynomial and graphs on surfaces


Reading: Lickorish ch 13-14, Prasolov-Sossinsky ch 8, Masbaum-Vogel, Ohtsuki Quantum Invariants ch 4.

8. Hyperbolic knots: Topics selected from: Hyperbolic geometry of 3-manifolds, figure-8 knot complement, ideal tetrahedra, gluing and completeness equations.

Reading:
- Purcell
- Adams
- Weeks


Reading:
- Ratcliffe: Foundations of hyperbolic manifolds chapter 10,
- Milnor
• Lackenby-Agol-DThurston
• Futer-Kalfagianni-Purcell
• SnaPy
• Guts of surfaces and the colored Jones polynomial chapter 9.

10. Relations of Quantum invariants to geometry: Open Questions and Progress.

Reading:

• H. Murakami: An Introduction to the Volume Conjecture
• Guts of surfaces and the colored Jones polynomial
• Jones polynomials, volume, and essential knot surfaces: A survey

Grade: Your grade in this course will be determined by attendance, participation in class discussions and questions, and a presentation or a written “take home” assignment.