CH. 4.3-4.4 (PART I). Logarithmic Function. Logarithmic equations.

- Change-of-Base Formula.

For any logarithmic bases $a$ and $b$, and any positive number $M$,

$$
\log _{b} M=\frac{\log _{a} M}{\log _{a} b}
$$

## Problem \#1.

Use your calculator to find the following logarithms. Show your work with Change-of-Base Formula.
a) $\log _{2} 10$
b) $\log _{1} 9$
c) $\log _{7} 11$

- Using the Change-of-Base Formula, we can graph Logarithmic Functions with an arbitrary base. Example:

$$
\begin{aligned}
& \log _{2} x=\frac{\ln x}{\ln 2} \\
& \log _{2} x=\frac{\log x}{\log 2}
\end{aligned}
$$



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- Properties of Logarithms.

If $b, M$, and $N$ are positive real numbers, $b \neq 1, p, x$ are real numbers, then

1. $\log _{b} M N=\log _{b} M+\log _{b} N$ product rule
2. $\log _{b} \frac{M}{N}=\log _{b} M-\log _{b} N \quad$ quotient rule
3. $\log _{b} M^{p}=p \log _{b} M \quad$ power rule
4. $\left\{\begin{array}{l}\log _{b} b^{x}=x \\ b^{\log _{b} x}=x, x>0\end{array} \quad\right.$ inverse property of logarithms
5. $\log _{b} M=\log _{b} N$ if and only if $M=N$. This property is the base for solving Logarithmic Equations in form $\log _{b} g(x)=\log _{b} h(x)$.

Properties 1-3 may be used for Expanding and Condensing Logarithmic expressions.

- Expanding and Condensing Logarithmic expressions.

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## Problem \#2.

Express each of the following expressions as a single logarithm whose coefficient is equal to 1 .
a) $\frac{1}{5}[3 \log (x+1)+2 \log (x-3)-\log 7]$
b) $\frac{1}{2}[\ln (x+1)+2 \ln (x-1)]+\frac{1}{3} \ln x$
c) $\frac{1}{2} \ln (x+3)-\frac{1}{5}[\ln x+3 \ln (x+1)]$
d) $\frac{1}{2}[\log (x-2)+2 \log (x+2)-\log 5]$

## Problem \#3.

Expand a much as possible each of the following.
a) $\log \frac{x^{2} y}{z^{5}}$
b) $\ln \sqrt[4]{\frac{x^{3} y}{z^{3}}}$

- Solving Logarithmic Equations.

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1. Solving the Simplest Logarithmic Equation (SLE). Given: $\log _{b} x=a, b>0, b \neq 1, a$ is any real number. According the definition of the logarithm this equation is equivalent to $x=b^{a}$.
2. According to properties of logarithms, if $\log _{b} M=\log _{b} N$, then $M=N$.

## Remember, check is part of solution for Logarithmic Equations.

Problem \#4. Solve the following Logarithmic Equations.
a) $\log _{2} x=5$
b) $\log _{3}(x-2)=5$
c) $\log \left(x^{2}-x\right)=\log 6$
d) $\log _{\frac{1}{2}}(x+4)=-3$

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e) $\log (x-15)=-2$
f) $\ln (x+3)=1$
g) $\log (2 x-1)=\log (x-2)$

