▪ Simple interest: concept and terminology.

Simple interest is a type of fee that is charged (or paid) *only* on the amount borrowed (or invested), and not on past interest.

Simple interest is generally used only on ***short-term*** notes – often on duration less than one year.

The amount invested (borrowed) is called the ***principal***. The ***interest*** (fee) is usually computed as a percentage of the principal (called the interest ***rate***) over a given period of time (unless otherwise stated, an ***annual rate***).

▪ Formulas for computing.

Simple interest is given by following formula:

$$I = Prt$$

I - interest, P - principal,

r - annual simple interst rate (in decimal form),

t - time in years.

When solving financial mathematics problems, **ALWAYS** specify all variables and their values.

Problem #1.

To buy furniture for a new apartment, Megan borrowed $4000 at 8% simple interest for 11 months. How much interest will she pay?

- Future or Maturity Value for Simple Interest.

Terminology.

If a principal P is borrowed at a rate r, then after t years the borrower will owe the lender an amount A that will include the principal P plus the interest I. Since P is the amount borrowed now and A is the amount that must be paid back in the future, P is often referred to as the *present value* and A as the *future value*. When loans are involved, the future value is often called the *maturity value* of the loan.

Formula relating A and P.

$$A = P + Prt = P(1 + rt)$$

We have four variables in this formula, A, P, r, t. Given any three variables, we can solve the equation for fourth.

- When solving financial mathematics problems, **ALWAYS** specify all variables and their values.

Problem #2. Future value.
That problem is similar to the Example #1, p.133 (PART II).

Find the maturity value for a loan of $2000 to be repaid in 6 months with interest of 9.4%.

Problem #3. Present Value of an Investment.
That problem is similar to the Example #2, p.133(PART II).

If you want to earn an annual rate of 15% on your investments, how much (to the nearest cent) should you pay for a note that will be worth $6,000 in 8 months?

Problem #4.

That problem is similar to the Example #3, p.134 (PART II).

Treasury bills(T-bills) are one of the instruments the U.S. Treasury Department uses to finance the public debt.
If you buy a 270-day T-bill with a maturity value of $10, 000 for $9,784.74, what annual interest rate will you earn? Express your answer as a percentage, correct to three decimal places. Use a 360-day year for simplicity of your computing.

> ***Note .***It is common to use for computing a 360-day year, 364-day year, 365-day year.

- **Recommendation.** Very useful and interesting are examples #4 and #5, pp 134-135 (PART II).

❖ Compound interest: concept and terminology.

As mentioned earlier (Lecture #21), *simple interest* is normally used for loans or investments of a year or less. For longer periods is used *compound interest*.
With *compound interest*, interest is paid on interest as well as on principal.

Problem #5.

$1000 is deposited at 3%.

a) What is the interest and the balance in the account at the end of the year?

b) If the amount is left at 3% interest for another year, what is the balance in the account at the end of second year?

❖ Formulas for computing.

- Compound amount with annual compounding period.

$A = P(1+r)^t$, where

P - principal (present value)

r - annual rate

t - time in years

A - amount in t years (future value)

- Compound amount with *m* compounding periods per year.

Interest can be compounded more that once per year:
semiannually(two period per year),
quarterly (four periods per year),
monthly (twelve periods per year),
daily (360, 364, or 365 periods per year)

The interest rate per period, *i*, can be found by dividing the annual interest rate, *r*, by the number of compound periods per year, *m*.
The total number of compounding periods, *n*, we can find by multiplying the number of years, *t*, by the number of compounding per year, *m*.

$A = P(1+i)^n$, where

A - future (maturity) value

P - principal (present value)

r - annual interest rate

m - number of compounding periods per year

t - time in years

n - total number of compounding periods

i - interest rate per period

$i = \frac{r}{m}$, $n = mt$

Note.

1. Each compound interest problem involves two rates:
a) the annual rate r;
b) the rate per compounding period, $i = \frac{r}{m}$.
You have to understand the distinction between them.
If interest is compounded annually, then $i = r$.

2. We have four variables in this formula, A, P, i, n.
Given any three variables, we can solve the equation for fourth.

3. When solving financial mathematics problems,
ALWAYS specify all variables and their values.

- Equivalent Formula for Compound Interest.

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

Problem #6. Computing Interest.

That problem is similar to the Example #1, pp.140-141, (PART II), textbook.

If $3000 is invested at 2.4% compounded quarterly, what interest you earn in 3 years?

The interest is the difference between future value and deposit (principal, present value).

$I = A - P$

Problem #7. Finding Present value.
That problem is similar to the Example #2, p.143, (PART II), textbook.

How much should you invest *now* at 5.5% compounded quarterly to have $6,000 toward the purchase of a car in 4 years?

Problem #8. Computing Growth Rate.
That problem is similar to the Example #3, p.144, (PART II), textbook.

If a $10,000 investment have grown to $126,000 in 10-year period, what annual rate compounded semiannually would produce this growth?

Problem #9. Growth time.
That problem is similar to the Example #4, p.145, (PART II), textbook.

How long it will take $15,000 to grow to $22,000 if it invested at 6% compounded monthly?

- Continuous Compounding
  (CH 4.1, pp 381-382, PART I).

Some banks use continuous compounding, there the number of compounding periods increases infinitely.
In such a case for computing is used the following formula.

After *t* years, the balance, *A*, in an account with principal *P* and annual interest rate *r* (in decimal form) is computed according to the following formula

$$A = Pe^{rt}$$

Problem #10. (Use the Problem #2 with the interest compounded continuously).

If $3000 is invested at 2.4% compounded continuously, what interest you earn in three years?
The interest is the difference between future value and deposit (principal, present value).

$$I = A - P$$

Problem #11. Computing Growth Rate.
(The Problem #4, but with continuous compounding).
If a $10,000 investment have grown to $126,000 in 10-year period, what annual rate compounded continuously would produce this growth?

Problem #12. Growth time.
(The Problem #5, but with continuous compounding).

How long it will take $15,000 to grow to $22,000 if it invested at 6% compounded continuously?