Simple interest: concept and terminology.

Simple interest is a type of fee that is charged (or paid) only on the amount borrowed (or invested), and not on past interest.

Simple interest is generally used only on short-term notes – often on duration less than one year.

The amount invested (borrowed) is called the principal. The interest (fee) is usually computed as a percentage of the principal (called the interest rate) over a given period of time (unless otherwise stated, an annual rate).

Formulas for computing.

Simple interest is given by following formula:

\[ I = Prt \]

\( I \) - interest, \( P \) - principal,
\( r \) - annual simple interest rate (in decimal form),
\( t \) - time in years.

When solving financial mathematics problems, ALWAYS specify all variables and their values.
Problem #1.

To buy furniture for a new apartment, Megan borrowed $4000 at 8% simple interest for 11 months. How much interest will she pay?

- Future or Maturity Value for Simple Interest.

Terminology.

If a principal $P$ is borrowed at a rate $r$, then after $t$ years the borrower will owe the lender an amount $A$ that will include the principal $P$ plus the interest $I$. Since $P$ is the amount borrowed now and $A$ is the amount that must be paid back in the future, $P$ is often referred to as the present value and $A$ as the future value. When loans are involved, the future value is often called the maturity value of the loan.

Formula relating $A$ and $P$.

$$ A = P + Prt = P(1 + rt) $$

We have four variables in this formula, $A, P, r, t$. Given any three variables, we can solve the equation for fourth.
When solving financial mathematics problems, **ALWAYS** specify all variables and their values.

**Problem #2.** Future value.
That problem is similar to the Example #1, p.133 (PART II).

Find the maturity value for a loan of $2000 to be repaid in 6 months with interest of 9.4%.

**Problem #3.** Present Value of an Investment.
That problem is similar to the Example #2, p.133(PART II).

If you want to earn an annual rate of 15% on your investments, how much (to the nearest cent) should you pay for a note that will be worth $6,000 in 8 months?
Problem #4.
That problem is similar to the Example #3, p.134 (PART II).

Treasury bills (T-bills) are one of the instruments the U.S. Treasury Department uses to finance the public debt. If you buy a 270-day T-bill with a maturity value of $10,000 for $9,784.74, what annual interest rate will you earn? Express your answer as a percentage, correct to three decimal places. Use a 360-day year for simplicity of your computing.

Note. It is common to use for computing a 360-day year, 364-day year, 365-day year.

- Recommendation. Very useful and interesting are examples #4 and #5, pp 134-135 (PART II).

✓ Compound interest: concept and terminology.

As mentioned earlier (Lecture #21), simple interest is normally used for loans or investments of a year or less. For longer periods is used compound interest. With compound interest, interest is paid on interest as well as on principal.
Problem #5.
$1000$ is deposited at $3\%$.

a) What is the interest and the balance in the account at the end of the year?

b) If the amount is left at $3\%$ interest for another year, what is the balance in the account at the end of second year?

Formulas for computing.
- Compound amount with annual compounding period.

\[
A = P(1 + r)^t, \text{ where}
\]
- $P$ - principal (present value)
- $r$ - annual rate
- $t$ - time in years
- $A$ - amount in $t$ years (future value)
- Compound amount with $m$ compounding periods per year.

Interest can be compounded more than once per year:
semiannually (two periods per year),
quarterly (four periods per year),
monthly (twelve periods per year),
daily (360, 364, or 365 periods per year)

The interest rate per period, $i$, can be found by dividing the annual interest rate, $r$, by the number of compound periods per year, $m$.
The total number of compounding periods, $n$, we can find by multiplying the number of years, $t$, by the number of compounding per year, $m$. 
\[ A = P \left(1 + i\right)^n, \quad \text{where} \]

- \(A\) - future (maturity) value
- \(P\) - principal (present value)
- \(r\) - annual interest rate
- \(m\) - number of compounding periods per year
- \(t\) - time in years
- \(n\) - total number of compounding periods
- \(i\) - interest rate per period

\[ i = \frac{r}{m}, \quad n = mt \]

**Note.**

1. Each compound interest problem involves two rates:
   a) the annual rate \(r\);
   b) the rate per compounding period, \(i = \frac{r}{m}\).

You have to understand the distinction between them. If interest is compounded annually, then \(i = r\).

2. We have four variables in this formula, \(A, P, i, n\).
   Given any three variables, we can solve the equation for fourth.

3. When solving financial mathematics problems, **ALWAYS** specify all variables and their values.
- Equivalent Formula for Compound Interest.

\[
A = P \left(1 + \frac{r}{m}\right)^{mt}
\]

**Problem #6. Computing Interest.**

That problem is similar to the Example #1, pp.140-141, (PART II), textbook.

If $3000 is invested at 2.4% compounded quarterly, what interest you earn in 3 years?

The interest is the difference between future value and deposit (principal, present value).

\[
I = A - P
\]
Problem #7. Finding Present value.
That problem is similar to the Example #2, p.143, (PART II), textbook.

How much should you invest *now* at 5.5% compounded quarterly to have $6,000 toward the purchase of a car in 4 years?

Problem #8. Computing Growth Rate.
That problem is similar to the Example #3, p.144, (PART II), textbook.

If a $10,000 investment have grown to $126,000 in 10-year period, what annual rate compounded semiannually would produce this growth?

Problem #9. Growth time.
That problem is similar to the Example #4, p.145, (PART II), textbook.

How long it will take $15,000 to grow to $22,000 if it invested at 6% compounded monthly?
Continuous Compounding
(CH 4.1, pp 381-382, PART I).
Some banks use continuous compounding, there the number of compounding periods increases infinitely.
In such a case for computing is used the following formula.

After $t$ years, the balance, $A$, in an account with principal $P$ and annual interest rate $r$ (in decimal form) is computed according to the following formula

$$A = Pe^{rt}$$
Problem #10. (Use the Problem #2 with the interest compounded continuously).

If $3000 is invested at 2.4% compounded continuously, what interest you earn in three years?
The interest is the difference between future value and deposit (principal, present value).
\[ I = A - P \]

Problem #11. Computing Growth Rate.
(The Problem #4, but with continuous compounding).
If a $10,000 investment have grown to $126,000 in 10-year period, what annual rate compounded continuously would produce this growth?

Problem #12. Growth time.
(The Problem #5, but with continuous compounding).

How long it will take $15,000 to grow to $22,000 if it invested at 6% compounded continuously?