- **<u>Linear Equation or first-degree equation in one variable.</u>**

- **<u>Definition.</u>** A linear or first-degree equation (LE) in one variable *x*
is an equation that can be written in the standard form

$$ax + b = 0,$$

where a and b are real numbers and $a \neq 0$

<u>Examples:</u>

$x = -1, \quad x - 2 = 6, \quad 3x + 5 = 0, \quad -5x + 7 = 3$

<u>Strategy for solving Linear Equations.</u> Isolate variable terms on one side and constant terms on the other side.

<u>Problem #1.</u> Solve the following Linear Equations.

a) $x - 2 = 6$ b) $3x + 5 = 0$ c) $-5x + 7 = 3$

Remember, to generate equivalents equations you can use the following operations.

1. Simplify an expression by removing grouping symbols and combining like terms.
2. Add /subtract the same real number/expression on both sides of the equation.
3. Multiply/divide on both sides of the equation by the same nonzero quantity
4. Interchange two sides of the equation.

Problem #2. Solve the following linear equation.

$$3\left[2-5(2x-4)\right]-5x=1-3x$$

• Equations that can be transformed into Linear Equations.

Equations with fractions.

To solve an equation with fractions we clear fractions first, means, multiply both on sides of the equation by LCD of all fractions and then reduce common factors in each fraction. The resulting equation does not contain fractions.

Problem #3.

a) $\frac{5}{x} - \frac{1}{3} = \frac{1}{2}$ b) $\frac{x}{x+7} = 8 - \frac{7}{x+7}$

Remember, when solving equations with fractions you have to make special assumptions to prevent the occurrence of ZEROS in the denominators. **“**Check**”** is MANDATORY for the equations which have unknown variable in denominators.

Linear inequalities.

$ax+b<c$, $ax+b\le c$, $ax+b>c$, $ax+b\ge c$.

To solve Linear Inequalities we use the same basic techniques used in solving Linear Equations.

> **Remember,** when both sides of an ***inequality*** are multiplied/divided by a ***negative*** number, ***the direction of the inequality symbol is reversed.***

We will write solutions for inequalities using the interval notation.

<u>Problem #4.</u>

Solve the following Linear Inequalities. Write your answers using the interval notation.

a) $2-3x>5$ b) $-3<1-2x\le 5$

Math 110 SS 2007
Lecture #2. Linear Equations and Inequalities. P1, 1.2, 1.7 (PART I)

- **Absolute Value of a real number. Definition.**

The absolute value of a real number a, denoted by $|a|$, is the distance from 0 to a on the number line.

The algebraic definition of the real number x (without reference to a number line):

$$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

Example 1. Evaluating Absolute Value.

a) $|5| = 5$; b) $|-2| = 2$; c) $|0| = 0$; d) $\left|\sqrt{2} - 1\right| = \sqrt{2} - 1$; e) $|2 - \pi| = \pi - 2$

- **Properties of Absolute Value.**

For all real numbers a and b,

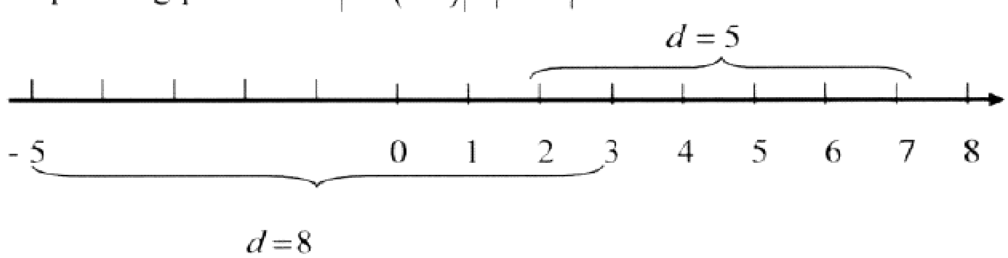
. $|a| \geq 0$ 2. $|-a| = |a|$ 3. $a \leq |a|$ 4. $|ab| = |a||b|$ 5. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$, $b \neq 0$

. $|a + b| \leq |a| + |b|$ (called the triangle inequality)

Distance between points on a Real Number Line.

For any two real numbers a and b, the distance between corresponding points on a number line is $|a - b|$.

Example 2. If $a = 3$, $b = -5$, then the distance on the number line between corresponding points $d = |3 - (-5)| = |3 + 5| = 8$.



If $a = 7$, $b = 2$, then the distance on the number line between corresponding points $d = |7 - 2| = |5|$.

- **<u>Linear Equations involving an Absolute Value.</u>**

Simplest Linear Equation containing an Absolute Value is $|x| = a,\ a \geq 0$.
To solve such equation means to find all real numbers x with the distance from the origin equal to a.

Example 3. The equation $|x| = 3$ has two solutions, $x = -3$ *and* $x = 3$, because there are two points on the number line whose distance from the origin equals 3.

- **<u>Linear inequalities involving an Absolute Value.</u>**

Simplest Linear Inequalities containing an Absolute Value are
$|x| > a,\ \ |x| \geq a,\ \ |x| < a,\ |x| \leq a.$
Example 4.

a) Solve the inequality $|x| \geq 4$. According to the definition of Absolute Value we need to find all points on a number line with the distance from the origin greater than 4 or equal to 4. Such points belong to the intervals $(-\infty, -4]$ *and* $[4, \infty)$. Answer: $(-\infty, -4] \cup [4, \infty)$.

b) Solve the inequality $|x| < 3$. According to the definition of Absolute Value we need to find all points on a number line with the distance from the origin is less then 3. Such points belong to the interval $(-3, 3)$.

c) Solve the inequality $|x| < -2$. This inequality has no solutions ($\varnothing$) because Absolute Value (distance) cannot be negative.

d) Solve the inequality $|x| > -1$. All real numbers satisfy this condition (each nonnegative number is greater than any negative one).
Solution set is $(-\infty, \infty)$.

- **<u>Practice.</u>**

1. Solve the following equations.
   a) $|x| = 0.5$ b) $|x| = \pi$ c) $|x| = 1000$ d) $|x| = -0.1$ e) $|2x| = 3$
2. Solve the following inequalities. Use the interval notation for your answers.
   a) $|x| \leq 3$ b) $|x| > -3$ c) $|x| < 101$ d) $|x| > \sqrt{3}$ e) $|x| \geq 0.01$.

<u>Note:</u> If in the given equation or inequality an absolute value is not isolated, then isolate it first.