Make sure you justify your statements completely and carefully.

1.

(a) Consider an injection φ of $\mathbb{F}_4$ into $\mathbb{F}_2^3$ with the property that

$$\varphi(\mathbb{F}_4) = \{001,\ 110,\ 010,\ 101\}\,.$$

Prove that, for any code $C \subseteq \mathbb{F}_4^n$, the corresponding expanded code $\hat{C}$ in $\mathbb{F}_2^{3n}$ has the property that each codeword has no more than three consecutive 0's and no more than three consecutive 1's among its entries. (This is a 'run-length-limited' constraint of the sort that is made for magnetic recording and on compact discs.)

(b) Prove that there are exactly four 4-subsets of $\mathbb{F}_2^3$ with the property discussed in (a).

2.

(a) For the linear code C over $F \geq K$, prove that C always contains $F \otimes_K (C|_K)$ but that the containment can be proper.

(b) For the linear code $D \leq K^n$ over $K \leq F$, prove

(i) $\dim_K D = \dim_F(F \otimes_K D)$

(ii) $D = (F \otimes_K D)|_K$

3. How many cyclic codes of length 8 over $\mathbb{F}_3$ are there? Give a generator polynomial for each such code.

4. Prove that there is no cyclic code that is (equivalent to) an $[8, 4]$ extended binary Hamming code.

5. Let cyclic code C have generator polynomial $g(x)$. Prove that C is contained in the sum-0 code if and only if $g(1) = 0$.

6. Give the cyclic generator matrix for the $[7, 4]$ binary cyclic code D with generator polynomial $x^3 + x^2 + 1$. Prove that D is a Hamming code.