Make sure you justify your statements completely and carefully.

1. For the polynomial $p(x) = \sum_{i=0}^{k} p_i x^i \in F[x]$, define the *formal derivative* of $p(x)$, denoted $p'(x)$, by $p'(x) = \sum_{i=1}^{k} i p_i x^{i-1}$.

(a) Let $a(x), b(x) \in F[x]$. Prove that $(a(x) + b(x))' = a'(x) + b'(x)$. Prove that $(a(x)b(x))' = a(x)b'(x) + a'(x)b(x)$.

(b) A polynomial $f(x)$ is *square free* in $F[x]$ if there are no nonconstant polynomials $g(x) \in F[x]$ for which $g(x)^2$ divides $f(x)$. Prove that $f(x)$ is square free if $\gcd(f(x), f'(x)) = 1$.

(c) Let $\alpha \in F$ be a root of $p(x) \in F[x]$. Prove that $(x - \alpha)^2$ divides $p(x)$ if and only if $x - \alpha$ divides $p'(x)$.

2. Problem 5.1.3 from the **Notes**.
(HINT: The thing to realize initially is that, for a given $\mathbf{c}$, we can have both $\mathbf{c} = \mathbf{ev}_{\alpha,v}(f(x)) \in \mathrm{GRS}_{n,k}(\boldsymbol{\alpha}, \mathbf{v})$ and $\mathbf{c} = \mathbf{ev}_{\beta,w}(g(x)) \in \mathrm{GRS}_{n,k}(\boldsymbol{\beta}, \mathbf{w})$ for different polynomials $f(x)$ and $g(x)$.)

For the final two problems, 'justifying your statements completely and carefully' means you should show your calculations in enough detail that I can follow them (and can try to locate any mistakes). ***Please do not do your calculations on a machine.*** *As these two problems are largely computational, I ask that you* ***do not collaborate*** *on them.*

3. Problem 5.2.5 from the **Notes**.

4. Consider the code $\mathrm{GRS}_{10,4}(\boldsymbol{\alpha}, \mathbf{v})$ over $\mathbb{F}_{11}$ that was our classroom example.
(a) Use Euclidean Algorithm decoding to decode the received vector

$$(0, 5, 9, 0, 1, 4, 5, 0, 6, 0).$$

(b) Use Euclidean Algorithm decoding to decode the received vector

$$(1, 1, 1, 1, 0, 0, 0, 0, 0, 0).$$