
Make sure you justify your statements completely and carefully.

1. Problem 3.1.9 from the **Notes**.

2. Consider the ternary code E of length 14 composed of those ternary words

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14})$$

that when arranged in an array as

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & & \\ x_4 & x_5 & x_6 & x_7 & \\ & x_8 & x_9 & x_{10} & x_{11} \\ & & x_{12} & x_{13} & x_{14} \end{array}$$

have each row and column summing to 0.

(a) Prove that E is a linear code.
(b) What is the dimension of E?
(c) What is the minimum distance of E?
(d) If the array

$$\begin{array}{ccccc} 0 & 0 & 1 & & \\ 0 & 0 & 2 & 0 & \\ & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 \end{array}$$

is received, give all possible decodings subject to **MDD**. That is, find all codewords (arrays) in E that are closest to this array.

3. Define the Hadamard product

$$\mathbf{x} * \mathbf{y} = (x_1 y_1, \ldots, x_n y_n)$$

for the vectors $\mathbf{x} = (x_1, \ldots, x_n)$ and $\mathbf{y} = (y_1, \ldots, y_n)$ of F^n. For instance, over $\mathbb{F}_{13}$,

$$(1, 2, 3, 4) * (2, 3, 4, 5) = (2, 6, 12, 7)\,.$$

(a) Prove: the dot product $\mathbf{c} \cdot \mathbf{d}$ is the sum of the entries in $\mathbf{c} * \mathbf{d}$, and in particular for binary $\mathbf{c}$ and $\mathbf{d}$ (in $\mathbb{F}_2^n$) we have $\mathbf{c} \cdot \mathbf{d} = \mathrm{w_H}(\mathbf{c} * \mathbf{d}) \pmod 2$.

(b) Prove that, for vectors $\mathbf{x} = (x_1, \ldots, x_n)$, $\mathbf{y} = (y_1, \ldots, y_n)$ in the vector space F^n over the field F, we have

$$\mathrm{w_H}(x + y) \geq \mathrm{w_H}(x) + \mathrm{w_H}(y) - 2\,\mathrm{w_H}(x * y)\,.$$

Prove additionally that, for the binary field $F = \mathbb{F}_2$, we have equality:

$$\mathrm{w_H}(x + y) = \mathrm{w_H}(x) + \mathrm{w_H}(y) - 2\,\mathrm{w_H}(x * y)\,.$$

4. Problem 3.1.11 from the **Notes**. (HINT: The last part of the previous problem might be of help in part (a).)

5. Problem 3.1.13 from the **Notes**.

6. (a) Give a syndrome dictionary for the $[8,4]$ binary code C with the following check matrix:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

(b) Use your dictionary to decode the received word:

$$(1,0,1,0,1,1,0,0).$$

(c) Use your dictionary to decode the received word:

$$(0,1,1,1,0,0,0,0).$$

(d) Use your dictionary to decode the received word:

$$(1,1,1,0,1,1,0,1).$$