You may talk with each other about problems but make sure you write up your solutions separately.

Make sure you explain things carefully.

Some remarks:

1. *Most problems that you will get this semester are not very difficult. Please try to solve the problems without looking them up in books, on the net, or whatever.*
2. *Examples and pictures can be of help in supporting or motivating an argument, but they do not constitute proofs.*
3. *"I wrote a computer program that said this." is* **not** *a valid proof.*
4. *Statements such as "it is trivial," "it is easy," "by a similar argument," and so forth will be treated with great skepticism.*

1. Problem 2.2.3 from the **Notes**:

`http://www.math.msu.edu/~jhall/classes/codenotes/coding-notes.html`

2. I have a 1-error-correcting code code C in $\{0,1\}^5$, and I know that there is no other 1-error-correcting code D in $\{0,1\}^5$ with $|D| > |C|$.

(a) Use the Sphere Packing Bound and the Gilbert-Varshamov Bound to find a lower and an upper bound on $|C|$.

(b) Give an example of a 1-error-correcting code E in $\{0,1\}^5$ with $|E| = |C|$. (Make sure you **prove** that your code has the largest possible size.)

3. Our Venn/Hamming code (from Class January 13 or Example 1.3.3 on page 12 of the **Notes**) is an example of 16 binary 7-tuples that form a 1-error-correcting code. We proved in class that it is impossible to find 16 binary 6-tuples that form a 1-error-correcting code. Indeed, we saw by the Sphere Packing Condition that a 1-error-correcting code in $\{0,1\}^6$ cannot have size bigger than 9.

(a) Prove that in fact there is no such code of size 9. (HINT: The previous problem may be of some help.)

(b) Find a 1-error-correcting code in $\{0,1\}^6$ of size 8. (HINT: Consider the Venn code.)

4. Let $\mathbb{P}$ be a DMC channel with input and output alphabet $A = \{0,1,2,3,4,5\}$ Consider the extended DMC channel $\mathbb{P}^{\otimes n}$ of length n.

(a) Assume that, in addition to $i \mapsto i$, the only *typical* symbol errors that can occur are

$$i \mapsto i-1 \pmod 6 \text{ and } i \mapsto i+1 \pmod 6\,.$$

Give a sphere packing bound for codes C in this situation, where $I = O = A^n$ and we wish to recover from *all typical errors.*

(b) Find a code C in $I = A^n$ that achieves your bound from (a).