Make sure you justify your statements completely and carefully.

1. Problem 5.1.3 from the Notes.
(HINT: The thing to realize initially is that, for a given \( c \), we can have both \( c = ev_{\alpha,v}(f(x)) \in GRS_{n,k}(\alpha,v) \) and \( c = ev_{\beta,w}(g(x)) \in GRS_{n,k}(\beta,w) \) for different polynomials \( f(x) \) and \( g(x) \).)

2. Problem A.2.26 from the Appendix 2 of the Notes. Prove first that
\[
(a(x) + b(x))' = a'(x) + b'(x).
\]

3. Problem 5.1.5 from the Notes. In parts (b) and (c) you may assume, for a finite field \( F \) with \( |F| = q \), that the set of all elements of \( F \) is precisely the set of roots of the polynomial \( x^q - x \) and, hence, that the nonzero elements of \( F \) are precisely the roots of the polynomial \( x^{q-1} - 1 \). (We will prove this later in the course.)

4. Let \( C \) and \( D \) be linear codes over the field \( F \) with \( C = D^\perp \). Let \( PC \) be the code \( C \) punctured at its last coordinate position, and let \( SD \) be the code \( D \) shortened at its last coordinate position. Prove that \( PC = SD^\perp \).

Remark. The code \( PC \) is constructed by deleting the last coordinate position from all codewords of \( C \). The code \( SD \) is constructed by first selecting only those codewords of \( D \) that end in 0, and then deleting from these that final 0 position.

For the final two problems, ‘justifying your statements completely and carefully’ means you should show your calculations in enough detail that I can follow them (and can try to locate any mistakes). Please do not do your calculations on a machine. As these two problems are largely computational, I ask that you not collaborate on them.

5. Problem 5.2.5 from the Notes.

6. Consider the code \( GRS_{10,4}(\alpha,v) \) over \( F_{11} \) that was our classroom example.
(a) Use Euclidean Algorithm decoding to decode the received vector
\[
(0,5,9,0,1,4,5,0,6,0).
\]
(b) Use Euclidean Algorithm decoding to decode the received vector
\[
(1,1,1,1,0,0,0,0,0,0).
\]