Make sure you justify your statements completely and carefully.

1. Problem 3.1.8 from the Notes.

2. Problem 3.1.9 from the Notes.

3. Define the Hadamard product \( x \ast y \) for the vectors \( x = (x_1, \ldots, x_n) \) and \( y = (y_1, \ldots, y_n) \) of \( F^n \). For instance, over \( F_{13} \),
\[
(1, 2, 3, 4) \ast (2, 3, 4, 5) = (2, 6, 12, 7).
\]

   (a) Prove: the dot product \( c \cdot d \) is the sum of the entries in \( c \ast d \), and in particular for binary \( c \) and \( d \) (in \( F_2^n \)) we have \( c \cdot d = w_H(c \ast d) \) (mod 2).

   (b) Prove that, for vectors \( x = (x_1, \ldots, x_n) \), \( y = (y_1, \ldots, y_n) \) in the vector space \( F^n \) over the field \( F \), we have
\[
w_H(x + y) \geq w_H(x) + w_H(y) - 2w_H(x \ast y).
\]

   Prove additionally that, for the binary field \( F = F_2 \), we have equality:
\[
w_H(x + y) = w_H(x) + w_H(y) - 2w_H(x \ast y).
\]

4. Problem 3.1.11 from the Notes. (Hint: The last part of the previous problem might be of help in part (a).)

5. (a) Give a syndrome dictionary for the \([8, 4]\) binary code \( C \) with the following check matrix:
\[
\begin{bmatrix}
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 
\end{bmatrix}
\]

   (b) Use your dictionary to decode the received word:
\[
(1, 0, 1, 0, 1, 0, 0, 0).
\]

   (c) Use your dictionary to decode the received word:
\[
(0, 1, 1, 1, 0, 0, 0, 0).
\]

   (d) Use your dictionary to decode the received word:
\[
(1, 1, 1, 0, 1, 0, 1, 0).
\]

6. Problem 4.1.5 from the Notes.

7. Problem 4.1.8 from the Notes.