

(1) Are the following statements true or false (circle one):

- a) e is a real number and $7 < 10$. [true — false]
- b) 119 is a prime number and $\sqrt{3}$ is a rational number. [true — false]
- c) 119 is not a prime number or $\sqrt{3}$ is a rational number. [true — false]
- d) $f(x) = e^x$ and $g(x) = |x|$ are differentiable at $x = 0$. [true — false]
- e) $f(x) = e^x$ or $g(x) = |x|$ are differentiable at $x = 0$. [true — false]

(2) Are the following statements true or false (circle one):

- a) If a and b are integers then $a + b$ is an integer. [true — false]
- b) If $\sum_{i=1}^{\infty} (-1)^i |a_i|$ converges, then $\sum_{i=1}^{\infty} |a_i|$ converges. [true — false]
- c) If f is continuous at $x = 0$, then f is differentiable there. [true — false]
- d) If $\lim_{i \rightarrow \infty} a_i = 0$, then $\sum_{i=1}^{\infty} a_i$ converges. [true — false]
- e) If $x > 5$ and $y > 5$, then $xy > 15$. [true — false]
- f) If $x > 5$ or $y > 5$, then $xy > 15$. [true — false]
- g) If squares have (only) three sides, then triangles have four sides. [true — false]

(3) Rephrase the following statements as 'if - then' statements:

- a) The differentiability of f is sufficient for f to be continuous.
- b) k is an even integer whenever $k + 1$ is odd.
- c) For all nonzero real numbers b the square b^2 is positive.
- d) $nm \geq 1$ provided that n and m are nonzero positive integers.
- e) For every positive real number ϵ there is a positive integer n so that $\frac{1}{n} < \epsilon$.

(4) a) Use truth tables to show that the following conditions are equivalent:

(A and B) and C – is equivalent to \neg A and (B and C)

A and (B or C) – is equivalent to \neg (A and B) or (A and C)

b) Show that the following sets are equal:

$$(S \cap T) \cap U = S \cap (T \cap U)$$

$$S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$$

(1) Use truth tables to show:

(a) $P \Rightarrow Q$ - is equivalent to $\neg(\neg Q \Rightarrow \neg P)$ (contrapositive).

(b) $\neg(P \Rightarrow Q)$ - is equivalent to $\neg P$ and $\neg Q$.

(c) $P \Rightarrow Q$ - is NOT equivalent to $\neg Q \Rightarrow P$

(2) Negate the following statements:

(a) e is a real number and $7 < 10$.

(b) 119 is a prime number and $\sqrt{3}$ is a rational number.

(c) 119 is not a prime number or $\sqrt{3}$ is a rational number.

(d) $f(x) = e^x$ and $g(x) = |x|$ are differentiable at $x = 0$.

(e) $f(x) = e^x$ or $g(x) = |x|$ are differentiable at $x = 0$.

(3) *State the contrapositive:*

(a) If a and b are integers then $a + b$ is an integer.

(b) If $\sum_{i=1}^{\infty} (-1)^i |a_i|$ converges, then $\sum_{i=1}^{\infty} |a_i|$ converges.

(c) If f is continuous at $x = 0$, then f is differentiable there.

(d) If $\lim_{i \rightarrow \infty} a_i = 0$, then $\sum_{i=1}^{\infty} a_i$ converges.

(e) If $x > 5$ and $y > 5$, then $xy > 15$.

(f) If $x > 5$ or $y > 5$, then $xy > 15$.

(g) If squares have (only) three sides, then triangles have four sides.