All answers must be justified appropriately.

From TREIL do the following exercises.
(Note that when we refer to Exercise a.b.c, we mean Exercise b.c from Chapter a.)

pages 72-73: 2.8.1, 2.8.2, 2.8.3, 2.8.5, 2.8.6
page 85: 3.3.1, 3.3.2,

As before, on **True/False** problem 2.8.1 "appropriate" justification is brief, say one or two sentences.

1. Let T be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined (in the standard coordinates $\mathcal{E}_2$) by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x+y \\ x-2y \end{pmatrix},$$

and let

$$\mathcal{A} = \left[\!\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right]\!\right] \quad \text{and} \quad \mathcal{B} = \left[\!\left[\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right]\!\right]$$

be two bases for $\mathbb{R}^2$.

(a) Find the matrix $[T]_{\mathcal{A},\mathcal{A}}$ of T with respect to the basis $\mathcal{A}$ in both the domain and target space of T.

(b) Find the matrix $[T]_{\mathcal{B},\mathcal{A}}$ of T with respect to the basis $\mathcal{A}$ in domain space and $\mathcal{B}$ in target space of T.

2. Let T, $\mathcal{A}$, and $\mathcal{B}$ be defined as in Problem 1 above. Find the matrix representing the change of coordinates from basis $\mathcal{A}$ to basis $\mathcal{B}$ in $\mathbb{R}^2$. Then show how to use this matrix to find your answer in 1(a) above from your answer in 1(b) above. (Verify that your answer in 1(a) is obtained in the process.)

3. Let $T : M_{2\times 2} \to \mathbb{R}^3$ be the linear transformation defined by

$$T(A) = \begin{pmatrix} 0 \\ \text{trace}(A) \\ 0 \end{pmatrix}.$$

Find the matrix $[T]_{\mathcal{E},\mathcal{M}}$ of T with respect to the basis

$$\mathcal{M} = [\![E_{1,1}\,,\, E_{1,2}\,,\, E_{2,1}\,,\, E_{2,2}]\!] = \left[\!\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right]\!\right]$$

for $M_{2\times 2}$ and the standard basis $\mathcal{E} = [\![\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]\!]$ for $\mathbb{R}^3$.