
All answers must be justified appropriately.

From TREIL do the following exercises.
(Note that when we refer to Exercise a.b.c, we mean Exercise b.c from Chapter a.)

page 66-67: 2.7.1, 2.7.2, 2.7.8
page 67: 2.7.3 (you need not find a basis for $\ker(A^\top)$)
page 68: 2.7.11 (*also* find a basis for the null space of the matrix), 2.7.12
page 67: 2.7.13, 2.7.14

As before, on **True/False** problem 2.7.1 "appropriate" justification is brief, say one or two sentences.

1. Let A be an $m \times n$ matrix with entries from the field $\mathbb{F}$. Prove that the linear transformation

$$T \colon \mathbb{F}^n \longrightarrow \mathbb{F}^m \text{ given by } T(\mathbf{v}) = A\mathbf{v}$$

is surjective if and only if the linear transformation

$$S \colon \mathbb{F}^m \longrightarrow \mathbb{F}^n \text{ given by } S(\mathbf{w}) = A^\top \mathbf{w}$$

is injective. HINT: Count pivots.

2. Consider the two linear transformations of finite dimensional $\mathbb{F}$-spaces

$$A \colon X \longrightarrow Y \quad \text{and} \quad B \colon W \longrightarrow X\,,$$

and the composition linear transformation $AB \colon W \longrightarrow Y$. Prove that

$$\operatorname{rank}(AB) \leq \min(\operatorname{rank} A, \operatorname{rank} B)\,.$$