All answers must be justified appropriately.

1. Chapter 1, Exercise 1.6 from the textnotes TREIL, page 5.
2. Let A and B be subspaces of the vector space V. Prove that their intersection
$$A \cap B = \{ z \mid z \in A,\ z \in B \}$$
is also a subspace of V.
3. Prove that the set
$$V = \{(x, y, z)^\top \in \mathbb{F}^3 \mid 3x + 5y + z = 0\}$$
is a subspace of $\mathbb{F}^3$.
4. Find a basis for the vector space V of the previous problem.
5. Let $\mathbb{P}_2$ denote the vector space of real polynomials of degree at most 2, that is, polynomials of the form
$$p(x) = a_2t^2 + a_1t + a_0.$$
Consider the following vectors in $\mathbb{P}_2$:
$$p_0(t) = 1$$
$$p_1(t) = t$$
$$p_2(t) = \frac{1}{2}(3t^2 - 1)$$
Show that these polynomials form a basis for $\mathbb{P}_2$. (These $p_i(t)$ are the first three of a famous sequence of polynomials called the *Legendre polynomials*. They may appear again later in the course.)
6. Chapter 1, Exercise 2.2 from the textnotes TREIL, page 11.
7. Chapter 1, Exercise 2.3 from the textnotes TREIL, page 11.
8. Show that for any value of θ, the vectors:
$$\mathbf{v_1} = \begin{pmatrix} \cos\theta \\ -\sin\theta \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$$
are linearly independent.
How are these vectors related to the standard basis vectors
$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad ?$$
9. Suppose $\{\mathbf{v_1}, \mathbf{v_2}\}$ is a basis for a vector space V. Let $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ be any set of vectors in V. Prove that $\{\mathbf{w_1}, \mathbf{w_2}, \mathbf{w_3}\}$ is not a basis for V.
10. Let
$$\{f : [0, 1] \to \mathbb{R}\}$$
denote the set of real-valued functions defined on the interval $[0, 1]$. This set is a vector space if we define addition and scalar multiplication by:
$$(f + g)(x) = f(x) + g(x), \quad \text{and} \quad (\lambda f)(x) = \lambda f(x),$$
respectively. The zero vector is given by the function whose value is zero at each $x \in [0, 1]$, i.e. $f(x) \equiv 0$. The additive inverse of a function $f(x)$ is given by $-f(x)$.
Prove that the vectors:
$$g(x) = x$$
$$h(x) = e^x$$
are linearly independent.