ON RESONANCES AND THE FORMATION OF GAPS IN THE SPECTRUM OF QUASI-PERIODIC SCHRÖDINGER EQUATIONS

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Abstract. We consider one-dimensional difference Schrödinger equations

\[ H(x, \omega)\varphi(n) \equiv -\varphi(n-1) - \varphi(n+1) + V(x + n\omega)\varphi(n) = E\varphi(n), \]

\[ n \in \mathbb{Z}, \quad x, \omega \in [0, 1] \] with real-analytic potential function \( V(x) \). If \( L(E, \omega_0) > 0 \) for all \( E \in (E', E'') \) and some Diophantine \( \omega_0 \), then the integrated density of states is absolutely continuous for almost every \( \omega \) close to \( \omega_0 \). In this work we establish the formation of a dense set of gaps in \( \text{spec}(H(x, \omega)) \cap (E', E'') \). Our approach is based on an induction on scales argument, and is therefore both constructive as well as quantitative. Resonances between eigenfunctions of one scale lead to “pre-gaps” at a larger scale. To pass to actual gaps in the spectrum, we show that these pre-gaps cannot be filled more than a finite (and uniformly bounded) number of times. To accomplish this, one relates a pre-gap to pairs of complex zeros of the Dirichlet determinants off the unit circle.