Math 828, Homework 4

Due October 4th

1. Exercise 1.3.8.

2. Exercise 1.3.13.

3. Exercise 1.3.17.

4. Exercise 1.3.18.

5. Exercise 1.3.19. Note: part of it was proved in the class but you still need to show complex linearity.

6. Exercise 1.3.22.

7. Exercise 1.3.23.

8. Exercise 1.3.24. Can one replace "pointwise almost everywhere" by "pointwise"? The last question is not mandatory for this homework, but is a nice exercise on cardinality of sets. We might get back to it later in the class.

9. Exercise 1.3.25. We used it in class but didn't do it in detail.

10. Let $f \colon [0,1] \to [0,1]$ be measurable. Define $f^*(x) = \lim\limits_{\varepsilon \to 0+} \sup\limits_{y \in (x-\varepsilon, x+\varepsilon)} f(y)$, $f_*(x) = \lim\limits_{\varepsilon \to 0+} \inf\limits_{y \in (x-\varepsilon, x+\varepsilon)} f(y)$. Show that f^* and f_* are measurable. Show that f is continuous at x if and only if $f^*(x) = f_*(x)$.