(20 points.)

True or false.

1. (2 Points) \( \int_1^\infty x^p \, dx \) converges if and only if \( p < -1 \).

2. (2 Points) \( \int_0^1 x^p \, dx \) converges if and only if \( p > -1 \).

3. (2 points) \( \lim_{n \to \infty} a_n = 0 \) is enough to guarantee \( \sum a_n \) converges.

4. (3 points) Let \( f(n) \) and \( g(n) \) be any nonzero polynomials in \( n \), and let \( \rho > 0 \). Then the following three series all have the same ratio test value, \( \rho \)

\[
\sum_{n=1}^{\infty} f(n)\rho^n, \quad \sum_{n=1}^{\infty} \frac{f(n)\rho^n}{g(n)}, \quad \sum_{n=1}^{\infty} \frac{\rho^n}{g(n)}.
\]

5. (2 points) The three series in the above problem have the same convergence statuses.
6. (2 points) Let $k \geq 2$ be an integer. The ratio test can be used to conclude that the following series converges

$$
\sum_{n=0}^{\infty} \frac{n!}{(n+k)!} = \frac{1}{k!} + \frac{1}{(k+1)!} + \frac{2}{(k+2)!} + \frac{3!}{(k+3)!} + \cdots
$$

7. (2 Points) Suppose $a_n > 0$ for all $n$. Then

$$
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} < 1 \implies \lim_{n \to \infty} a_n = 0.
$$

8. (2 points) The series $\sum_{n=1}^{\infty} \cos(n)/n$ is convergent, but not absolutely.

9. (3 points) Let $x \in [0, \pi/2)$. Then $\sum_{n=0}^{\infty} (\sin x)^{2n} = \sec^2 x$. 