1. (2 points) Use Newton’s method (once) to estimate the value of $\sqrt{5}$.

**Solution.** The function $f(x) = x^2 - 5$ has $\sqrt{5}$ as a root. And $\sqrt{5}$ is close to the number 2 (because $2^2 = 4$ and $3^2 = 9$, so it is closer to 2 than it is to 3.) Therefore we use $x_1 = 2$ and we use Newton’s method to approximate

$$\sqrt{5} \approx x_1 - \frac{f(x)}{f'(x)} = 2 - \frac{2^2 - 5}{2 \cdot 2} = 2.25.$$ 

2. (1+1+2=4 points) Find $f$ in each case.

(a) $f'(x) = px^{p-1}, \ p \neq 0, \ f(0) = 1.$

(b) $f'(x) = \pi \cos(\pi x), \ f(1) = 0.$

(c) $f'(x) = 1 - \csc(x) \cot(x).$

**Solution.**

(a) $f(x) = x^p + 1.$

(b) $f(x) = \sin(\pi x).$

(c) $f(x) = x + \csc x + C.$

**Solution.**

3. (2+1=3 points) (a) Using 8 rectangles of equal width, and using the left endpoints of each rectangle, *estimate* the area under the graph of $y = \sqrt{x}$ from $x = 1$ to $x = 5$. (b) Is the estimate an over estimate or underestimate?

**Solution.**

(a) We are told to use $n = 8$ rectangles, and we calculate that they all have width $\Delta x = \frac{5-1}{8} = \frac{1}{2}$. The endpoints of the rectangles occur at the numbers $x_k = 1 + k \cdot \Delta x = 1 + \frac{k}{2}$ for $k = 0, ..., 8$. (The $i$-th rectangle has left endpoint $x_{i-1}$ and right endpoint $x_{i}$.) Since we are using left endpoints, we choose our sample points as $x^*_k = x_{k-1}$. Recall
that the “sample points” are the numbers we plug into \( y = f(x) \) to get the height of each rectangle. Therefore the area \( A \) under the curve has the left hand approximation

\[
L_8 = \sum_{k=1}^{8} f(x_k^*) \Delta x = \sum_{k=1}^{8} \left( \sqrt{x_{k-1}} \cdot \frac{1}{2} \right) = \frac{1}{2} \sum_{k=1}^{8} \sqrt{1 + \frac{k-1}{2}}.
\]

Comment. You can also write the above sum as \( \frac{1}{2} \sum_{k=0}^{7} \sqrt{1 + \frac{k}{2}} \).

(b) It is an underestimate because \( y = \sqrt{x} \) is monotonically increasing, i.e., \( y' \geq 0 \).