1. (1+1+1+1=4 points) Let \( f'(x) = 1 - x^2 \) be the derivative of a differentiable function \( f \).
   (i) Where is \( f \) increasing/decreasing?
   (ii) Where do the local minima/maxima of \( f \) occur?
   (iii) Describe the concavity of \( f \).
   (iv) Suppose \( f(0) = 0 \). Sketch a graph of \( f \).

Solution

(i) \( f \) is increasing on \([-1, 1]\) and decreasing on \((\infty, -1] \cup [1, \infty)\).
(ii) The \( x = -1 \) is a local minimizer and \( x = 1 \) is a local maximizer.
(iii) Since \( f''(x) = -2x \) has a zero at 0, and is negative for \( x > 1 \) and positive for \( x < 1 \) we see that \( f \) is concave up on \((\infty, 0)\) and concave down on \((0, \infty)\).
(iv) We recently learned about anti-derivatives, so you should know that \( f(x) = x - \frac{1}{3}x^3 + C \). But since \( f(0) = 0 \), we have \( C = 0 \). Go to www.google.com and type “Plot \( y = x - x^3/3 \)” to check whether your graph is correct. I personally find Google even better than Wolfram for plotting, because you can zoom.

2. (6 points) Suppose you are given 12 ft\(^2\) of material with which to build a square-bottom, open-top rectangular box. Determine the dimensions of the box with the largest possible volume.

Answer: \( x = 2, y = 1 \).

Note: you obviously need to show work on an exam. This is just an answer key.