1. (2 points) Use a linear approximation to estimate $\tan(47^\circ)$.

(Simplify your answer.)

**Solution.** Observe that $47^\circ = \frac{47\pi}{180} = \frac{\pi}{4} + \frac{\pi}{90}$. Since we know how to evaluate both $\tan(x)$ and $\tan'(x) = \sec^2(x)$ at $a = \pi/4$, that is where we will center our approximation. From the linear approximation formula, we then have:

$$\tan\left(\frac{47\pi}{180}\right) \approx \tan'(a)\Delta x + \tan(a) = \sec^2\left(\frac{\pi}{4}\right) \cdot \left(\frac{\pi}{90}\right) + \tan\left(\frac{\pi}{4}\right) = \frac{\pi}{45} + 1.$$  

2. (3 points) Find all of the minima and maxima of the polynomial $f(x) = 2x^3 - 15x^2 + 36x$ on the interval $[0, 4]$, and label which ones are local minima/maxima and label which ones are global minima/maxima.

**Solution.** $f'(x) = 6x^2 - 15x + 36 = 6(x-2)(x-3)$. Therefore the critical points are $x = 2$ and $x = 3$. In order to determine the global and local minima/maxima, we need to compare the values of $f(0), f(2), f(3)$ and $f(4)$. We can easily see that $f(0) = 0$.

Next, we see that $f'(x)$ is positive on $(0, 2)$ and negative on $(2, 3)$ and positive on $(3, 4)$. So the only two candidates for a global maximizer are $x = 2$ and $x = 4$, however $f(4) = 32 > f(2) = 28$ so we conclude that $f(2)$ is a local maximum and $f(4)$ is the global maximum. The only two candidates for a global minimizer are $x = 0$ and $x = 3$. However, $f(3) = 27 > f(0) = 0$ so we conclude that $f(0) = 0$ is the global minimum, and $f(3)$ is a local minimum.

3. (a) (3 points) State the mean value theorem.

**Solution.** The Mean Value Theorem: If $f$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval $(a,b)$, then there exists a number $c \in (a,b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$
(b) (2 points) Use the mean value theorem to prove that

\[ 3x - \sin x = 0 \]

has exactly one solution (namely \( x = 0 \)).

Solution. Suppose there were two solutions \( a \) and \( b \) (say \( a < b \)) then the mean value theorem asserts that there exists a number \( c \) between \( a \) and \( b \) such that

\[ 0 = \frac{f(b) - f(a)}{b - a} = f'(c) = 3 - \cos c, \]

which is impossible, since \( \cos x \) never equals 3.