Math 132.035, Quiz 2 Solutions
9/12/14 (10 points, 20 minutes)

1. (a) (1 point) Let $f(x)$ be a function. Give the $\delta, \epsilon$-definition of the limit of $f$ as $x$ approaches $a$

Solution:

$$
\lim_{x \to a} f(x) = L \iff \forall \epsilon > 0, \exists \delta > 0 \text{ such that } |x-a| < \delta \implies |f(x)-L| < \epsilon
$$

(b) (3 points) Using the $\delta, \epsilon$-definition, prove the following limit

$$
\lim_{x \to 2} x^2 - 4x + 10 = 6
$$

(That is, given $\epsilon > 0$, your goal is to find $\delta > 0$ satisfying the definition of the limit.)

Solution: Let $\epsilon > 0$ be an arbitrary positive number.

$$
|x^2 - 4x + 10 - 6| < \epsilon
\iff |(x-2)^2| < \epsilon
\iff |x-2| < \sqrt{\epsilon}
$$

Choosing $\delta = \sqrt{\epsilon}$ therefore satisfies the definiton of the limit.
2. (3 points) Determine the value of \( k \) for which the following function is continuous.

\[
f(x) = \begin{cases} 
\frac{2x^3 + x^2 + x + 2}{x + 1} & \text{for } x > -1 \\
2x^2 - x + k & \text{for } x \leq -1 
\end{cases}
\]

\textbf{Solution:} The component functions are both continuous (except for maybe at } x = -1\text{, so we may force the total function } f \text{ to be continuous by forcing the components to both approach the same limit as } x \to -1\text{, which we can do by choosing the correct } k. \text{ Suppose } x \neq -1 \text{ then}

\[
2x^2 - x + k = \frac{(2x^2 - x + k)(x + 1)}{x + 1} = \frac{2x^3 - x^2 + kx + 2x^2 - x + k}{x + 1}
\]

\[
= \frac{2x^3 + x^2 + (k - 1)x + k}{x + 1}.
\]

Compare this to the first component function of } f. \text{ Choosing } k = 2 \text{ will then force the components to have the same limit as } x \to -1.
3. (3 points) Let \( f(x) = \sqrt{1-x} \). Using the definition of the derivative, compute \( f'(-3) \).

**Solution:**

\[
f'(x) = \lim_{z \to x} \frac{\sqrt{1-z} - \sqrt{1-x}}{z-x}
\]

(multiply by the conjugate)

\[
= \lim_{z \to x} \frac{(1-z) - (1-x)}{(z-x)(\sqrt{1-z} + \sqrt{1-x})}
\]

\[
= \lim_{z \to x} \frac{x-z}{(z-x)(\sqrt{1-z} + \sqrt{1-x})}
\]

\[
= \lim_{z \to x} \frac{-1}{\sqrt{1-z} + \sqrt{1-x}} = -\frac{1}{2\sqrt{1-x}}.
\]

So \( f'(-3) = -1/4 \).

**Remark:** We can also do this derivative using derivative rules (i.e. the power rule + the chain rule) to verify our computation

\[
f'(x) = ((1-x)^{1/2})' = \frac{1}{2}(1-x)^{-1/2}(-1)
\]