Math 132.035, Quiz 1 Solutions
9/5/14 (20 points, 20 minutes)

1. Let \( f(x) = x^2 + 2x \).

   (a) (1 point) Find the slope of the secant line joining the points \( P(1, f(1)) \) and \( Q(2, f(2)) \).

   Solution: 
   \[
   \frac{(2^2 + 2 \cdot 2) - (1^2 + 2 \cdot 1)}{2 - 1} = \frac{5}{1} = 5
   \]

   (b) (1 point) Find the slope of the secant line joining the points \( P(1, f(1)) \) and \( Q(1 + h, f(1 + h)) \).

   Solution: 
   \[
   \frac{(1 + h)^2 + 2(1 + h)) - (1^2 + 2 \cdot 1)}{(1 + h) - h} = \frac{(h^2 + 4h + 3) - 3}{h} = h + 4
   \]

   (c) (2 points) Take a limit of the above slope as \( h \to 0 \) in order to find the slope of the tangent line at \( P(1, f(1)) \).

   Solution: 
   \[
   \lim_{h \to 0} h + 4 = 4
   \]

   (d) (1 point) Using your answer from the previous part, give an equation for the tangent line at \( P(1, f(1)) \).

   Solution: The point slope equation
   \[
   y - y_0 = m(x - x_0)
   \]
   gives
   \[
   y - 3 = 4(x - 1),
   \]
   Which simplifies to
   \[
   y = 4x - 1
   \]
2. (1 pt each) Evaluate each of the following expressions. Write DNE if the limit or function evaluation does not exist.

a) \( \lim_{x \to -1^-} F(x) = -3 \)

b) \( \lim_{x \to -1^+} F(x) = -3 \)

c) \( \lim_{x \to -1} F(x) = -3 \)

d) \( F(-1) = -2 \)

e) \( \lim_{x \to 1^-} F(x) = 2 \)

f) \( \lim_{x \to 1^+} F(x) = 3 \)

g) \( \lim_{x \to 1} F(x) \quad \text{DNE} \)

h) \( \lim_{x \to 3} F(x) = 0 \)

i) \( F(3) \quad \text{DNE} \)
3.

4. (3 points)

\[ \lim_{h \to 0} \frac{7(1 + h)^2 - 7}{h} \]

Solution:

\[ = \lim_{h \to 0} \frac{7h^2 + 14h + 7 - 7}{h} = \lim_{h \to 0} 7h + 14 = 14 \]

Remark: If you let \( f(x) = 7x^2 \) then \( f'(x) = 14x \) so that

\( f'(1) = 14. \)

Which is good, since the above limit computation is simply the computation of \( f'(1) \) via the definition.

5. (3 points)

\[ \lim_{h \to 0} \frac{1}{h+2} - \frac{1}{2} \]

Solution:

\[ = \lim_{h \to 0} \frac{2 - (2 + h)}{2h(2 + h)} = \lim_{h \to 0} \frac{-1}{2(2 + h)} = \]

\[ = -\frac{1}{4} \]

Remark: If you let

\( f(x) = \frac{1}{x} \)

then the above limit computation is simply the computation of \( f'(2) \) which is done below, using derivative rules.

\[ f'(2) = \left( \frac{1}{x} \right)'_{x=2} = (x^{-1})'_{x=2} = \]

\[ = -x^{-2}_{x=2} = -\frac{1}{4} \]