1. Use definition to find the Taylor series of the function and prove convergence for all $x \in \mathbb{R}$ by showing $\lim_{n \to \infty} R_n(x) = 0$.
   (a) $f(x) = \cos x$
   (b) $f(x) = \sinh x = \frac{e^x - e^{-x}}{2}$  
   (This is Exercise 31.1 and part of 31.2.)

2. Use definition to find the Taylor series of the function and the Lagrange form of the remainder.
   (a) $f(x) = (1 + x)^{1/3}$
   (b) $f(x) = \frac{1}{\sqrt{1 + x}}$
   (These are the special case of Theorem 31.7 for $\alpha = 1/3$ and $\alpha = -1/2$. The remainder used there is in the Cauchy form, not the Lagrange form.)

3. (1) Write $f(x) = e^x$ in the form $f(x) = f_n(x) + R_n(x)$ for $c = 0$ and $n = 3$, where $f_n(x) = \sum_{k=0}^{n-1} \frac{f^{(k)}(c)}{k!} (x - c)^k$ is the Taylor Polynomial of $f(x)$ of order $n$ and $R_n(x)$ is the remainder in the Lagrange form.
   (b) Use the result you get from part (a) to find $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$.

4. Repeat Exercise 3 for $f(x) = \sqrt{1 - x}$ and $n = 3$, and then find $\lim_{x \to 0} \frac{\sqrt{1 - x} - 1 + \frac{x}{2}}{x^2}$.

5. Repeat Exercise 3 for $f(x) = \cos x$ and $n = 5$, and then find $\lim_{x \to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{6x^4}$.