24. (Statement of the problem.)

Proof. a) Since A is row equivalent to B, by the definition, there are elementary matrices E_1, \dots, E_k such that

$$A = E_k \cdots E_1 B. \tag{1}$$

Since B is row equivalent to C, by the definition, there are elementary matrices F_1, \dots, F_j (Note: Use different notation!) such that

$$B = F_i \cdots F_1 C. \tag{2}$$

Substitute (2) into (1) we get

$$A = E_k \cdots E_1 F_j \cdots F_1 C.$$

Since $E_1, \dots, E_k, F_1, \dots, F_j$ are elementary matrices, by the definition, A is row equivalent to C.

b) Suppose both A and B are nonsingular $n \times n$ matrices. By Theorem 1.4.2, A is row equivalent to I and B is row equivalent to I. Since B is row equivalent to I, by Problem 23, I is row equivalent to B. So we have that A is row equivalent to I and I is row equivalent to B. By Part (a), we get that A is row equivalent to B. \Box

By Problem 23, I is row equivalent to B.

A short version of Part (b) is like the following:

Suppose both A and B are nonsingular $n \times n$ matrices.

By Theorem 1.4.2, A is row equivalent to I and B is row equivalent to I.

By Part (a), A is row equivalent to B. \Box