

26. Let A be an $m \times n$ matrix.

(b) Show that $A^T A$ and AA^T are both symmetric.

Proof.

$$\begin{aligned}(A^T A)^T &= A^T (A^T)^T && \text{(By Algebraic Rule 4 for Transpose)} \\ &= A^T A. && \text{(By Algebraic Rule 1 for Transpose)}\end{aligned}$$

By the definition of symmetry, $A^T A$ is symmetric.

$$\begin{aligned}(AA^T)^T &= (A^T)^T A^T && \text{(By Algebraic Rule 4 for Transpose)} \\ &= AA^T. && \text{(By Algebraic Rule 1 for Transpose)}\end{aligned}$$

By the definition of symmetry, AA^T is symmetric. \square