Recall that $\int_0^{2\pi} \sin^2 t\,dt = \pi$. Now let n be a positive integer. Then

$$\int_0^{2\pi} \sin^2 nt\,dt = \pi$$

To see this, let $u = nt$. Then $du = n\,dt$ and

$$\begin{aligned}\int_0^{2\pi} \sin^2 nt\,dt &= \frac{1}{n}\int_0^{2\pi n} \sin^2 u\,du \\ &= \frac{1}{n}\left(\int_0^{2\pi} + \int_{2\pi}^{4\pi} + \cdots + \int_{2\pi(n-1)}^{2\pi n} \sin^2 u\,du\right) \\ &= \frac{1}{n}\sum_{j=1}^{n}\int_{2\pi(j-1)}^{2\pi j} \sin^2 u\,du \\ &= \frac{1}{n}\sum_{j=1}^{n}\pi \\ &= \frac{1}{n}(n\cdot\pi) \\ &= \pi\end{aligned}$$

In particular,

$$\int_0^{2\pi} \sin^2 2t\,dt = \pi$$

During Friday's review I mistakenly stated that the last integral was equal to 2π. It now follows that

$$\begin{aligned}\iint_{S_2} \mathbf{F}\cdot\mathbf{n}\,dS &= -\int_0^{2\pi}\int_0^1 (3\cos^3 t + \sin^2 t\,\cos^2 t)\,ds\,dt \\ &= -\int_0^{2\pi} (3\cos^3 t + \sin^2 t\,\cos^2 t)\,dt \\ &= -3\underbrace{\int_0^{2\pi} \cos^3 t\,dt}_{0} - \int_0^{2\pi} \sin^2 t\,\cos^2 t\,dt \\ &= \frac{-1}{4}\int_0^{2\pi} \sin^2 2t\,dt \\ &= \frac{-\pi}{4}\end{aligned}$$

as expected.