1. (2 points) Let $F(x) = \int_x^0 \sqrt{1-t^2}\,dt,\ x \in [-1,1]$. Find $F'(1/2)$.

   Solution:

   According to the Fundamental Theorem of Calculus

$$\begin{aligned} F'(x) &= D_x\left(\int_x^0 \sqrt{1-t^2}\,dt\right) \\ &= D_x\left(-\int_0^x \sqrt{1-t^2}\,dt\right) \\ &= -\sqrt{1-x^2} \end{aligned}$$

   It follows that

$$F'(1/2) = -\frac{\sqrt{3}}{2}$$

2. (3 points) Find the average value of $g(x) = x^2$ over the interval $[-1,3]$.

   Solution:

$$g_{\text{avg}} = \frac{1}{3-(-1)}\int_{-1}^{3} x^2\,dx = \frac{1}{12}x^3\bigg|_{-1}^{3} = \frac{7}{3}$$

3. (5 points) Evaluate the definite integral below.

$$\int_0^{\pi/4} \frac{dx}{\cos^2 x\,\sqrt{1+\tan x}}$$

Solution:

Let $u = 1 + \tan x$. Then $du = \sec^2 x\,dx$ and

$$\begin{aligned}\int_0^{\pi/4} \frac{dx}{\cos^2 x\,\sqrt{1+\tan x}} &= \int_0^{\pi/4} \frac{\sec^2 x\,dx}{\sqrt{1+\tan x}} \\ &= \int_{u(0)}^{u(\pi/4)} \frac{du}{\sqrt{u}} \\ &= 2\sqrt{u}\,\Big|_1^2 \\ &= 2\left(\sqrt{2}-1\right)\end{aligned}$$