1. (4 points) Solve the following initial value problem.

$$\frac{dy}{dx} = \frac{1}{\sqrt{1+2x}}, \quad y(0) = 3$$

Solution:

Using the integral sign notation as we did in class we see that the differential equation above implies

$$y = \int \frac{dx}{\sqrt{1+2x}}$$

We must find a function whose derivative is $1/\sqrt{1+2x}$. Our first guess is to let

$$y = \sqrt{1+2x} + C$$

That turns out to be correct since

$$\frac{dy}{dx} = D\left(\sqrt{1+2x}\right) = \frac{1}{2\sqrt{1+2x}} \times 2$$

So $y(x) = \sqrt{1+2x} + C$. Now since

$$3 = y(0) = \sqrt{1+2(0)} + C \quad \Longrightarrow \quad C = 2$$

It follows that

$$y = \sqrt{1+2x} + 2$$

2. (6 points) In each case, find the most general antiderivative. *Check your work!*

(a) $\displaystyle\int x \sin x^2 \, dx = \frac{-\cos x^2}{2} + C$

Check:

$$D_x\left(\frac{-\cos x^2}{2}\right) = \frac{-1}{2} D_x\left(\cos x^2\right) = \frac{-1}{2}(-\sin x^2)(2x) = x \sin x^2$$

(b) $\displaystyle\int \left(\frac{12}{x^4} + \frac{3x^2}{4}\right) dx =$

Solution:

$$\begin{aligned}
&= \int \frac{12}{x^4}\, dx + \int \frac{3x^2}{4}\, dx \\
&= 12 \int x^{-4}\, dx + \frac{3}{4} \int x^2\, dx \\
&= 12 \frac{x^{-4+1}}{-4+1} + \frac{3}{4}\frac{x^3}{3} \\
&= \frac{-4}{x^3} + \frac{x^3}{4} + C
\end{aligned}$$

Check:

$$\begin{aligned}
D_x\left(\frac{-4}{x^3} + \frac{x^3}{4}\right) &= -4 D_x\left(x^{-3}\right) + \frac{1}{4} D_x\left(x^3\right) \\
&= -4(-3x^{-3-1}) + \frac{1}{4}\left(3x^2\right) \\
&= \frac{12}{x^4} + \frac{3x^2}{4}
\end{aligned}$$