1. (5 points) Let $f(x) = x^3 - \dfrac{15}{2}x^2 + 12x + 1$. Find the absolute maximum value of $f(x)$ on the interval $[2, 5]$ and indicate where it occurs.

Solution:

$$f'(x) = 3x^2 - 15x + 12 = 3(x-1)(x-4)$$

So the critical points are $x = 1, 4$. Notice that $1 \notin [2, 5]$. Now choose the largest of

$$\begin{aligned} f(2) &= 3 \\ f(4) &= -7 \\ f(5) &= -3/2 \end{aligned}$$

So the absolute maximum is $f(2) = 3$.

2. (5 points) Use linear approximation (or differentials) to estimate $\sqrt{16.4}$.

Solution:

Let $g(x) = \sqrt{x}$ and notice that $g(16)$ is an integer. So we will try to estimate $g(16.4)$ by finding the linearization of $g(x)$ at 16. Thus

$$\begin{aligned} L(x) &= g(16) + g'(16)(x - 16) \\ &= 4 + \frac{1}{2\sqrt{16}}(x - 16) \end{aligned}$$

It follows that

$$\sqrt{16.4} \approx L(16.4) = 4 + \frac{1}{8}(16.4 - 16) = \frac{81}{20}$$

It is worth observing that

$$\left(\frac{81}{20}\right)^2 = 16.4025$$

So the estimate is pretty good.